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Priest's Motorbike and Tolerant Identity

Pablo Cobreros, Paul Egré, David Ripley and Robert van Rooij

Abstract In his chapter 'Non-transitive identity' [8], Graham Priest develops a notion of non-transitive identity based on a second-order version of LP. Though we are sympathetic to Priest's general approach to identity we think that the account can be refined in different ways. Here we present two such ways and discuss their appropriateness for a metaphysical reading of indefiniteness in connection to Evans' argument.

Keywords Logical consequence · Identity · Indeterminacy · Logic of paradox

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1 Priest's Motorbike and LP-Identity

1.1 Priest's Motorbike

Priest motivates his account of identity based on the following case:

> Suppose I change the exhaust pipes on my bike; is it or is it not the same bike as before? It is, as the traffic registration department and the insurance company will testify; but it is not, since it is manifestly different in appearance, sound, and acceleration.
>
> Dialecticians, such as Hegel, have delighted in such considerations, since they appear to show that the bike both is and is not the same. A standard reply here is to distinguish between the bike itself and its properties. After the change of exhaust pipes the bike is numerically the same bike; it is just that some of its properties are different. Perhaps, for the case at hand, this is the right thing to say. But the categorical distinction between the thing itself and its properties is one which is difficult to sustain; to suppose that the bike is something over and above all of its properties is simply to make it a mysterious *Ding an sich*. Thus, suppose that I change, not just the exhaust pipes, but, in succeeding weeks, the handle bars, wheels, engine, and in fact all the parts, until nothing of the original is left. It is now a numerically different bike, as even the traffic office and the insurance company will concur. At some stage, it has changed into a different bike, i.e. it has become a different machine: the bike itself is numerically different. [8, 406]

Other cases of this sort seem to show that identity fails to be transitive. There is an implicit link in the literature between the ideas that identity is transitive and that indeterminacy associated to vagueness is purely semantic. As David Lewis puts it:

> The reason it's vague where the outback begins is not that there's this thing, the outback, with imprecise borders; rather there are many things, with different borders, and nobody has been fool enough to try to enforce a choice of one of them as the official referent of the word 'outback'. Vagueness is semantic indecision [6, 213].

In the following section we review Priest's strategy to define a non-transitive notion of identity based on LP.

1.2 Second-Order LP and Identity

LP is a paraconsistent logic with a natural dialetheist interpretation: for some property P and thing a, a is both P and not-P. We can formulate LP's semantics in a very straightforward way making use of three values and a Strong-Kleene valuation schema (our presentation is different in style, but equivalent to Priest's [8]). More specifically, for a first-order language (just unary predicates and no complex terms) $\mathscr{L}$:

Definition 1 An *MV-model* is a structure $\langle D, \mathbb{I}\rangle$ such that:

- D a non-empty domain of quantification.
- $\mathbb{I}$ is an interpretation function:

– For a name or variable a, $\mathbb{I}(a) \in D$
– For any predicate P, $\mathbb{I}(P) \in \{1, \frac{1}{2}, 0\}^D$
– For an atomic formula Pa, $\mathbb{I}(Pa) = \mathbb{I}(P)\mathbb{I}(a)$
– $\mathbb{I}(\neg A) = 1 - \mathbb{I}(A)$
– $\mathbb{I}(A \wedge B) = min(\mathbb{I}(A), \mathbb{I}(B))$
– $\mathbb{I}(A \vee B) = max(\mathbb{I}(A), \mathbb{I}(B))$
– $\mathbb{I}(\exists x A) = max(\{\mathbb{I}'(A) : \mathbb{I}'$ is an x-variant of $\mathbb{I}\})$
– $\mathbb{I}(\forall x A) = min(\{\mathbb{I}'(A) : \mathbb{I}'$ is an x-variant of $\mathbb{I}\})$

Definition 2 We say that $\Gamma \vDash^{LP} \Delta$ iff there is no MV-model M such that $\mathbb{I}(A) > 0$, for every $A \in \Gamma$ and $\mathbb{I}(B) = 0$ for every $B \in \Delta$.

The material conditional $(A \supset B)$ is defined as $(\neg A \vee B)$ and the material biconditional $(A \equiv B)$ as $(A \supset B) \wedge (B \supset A)$.

Consider now the expansion of $\mathscr{L}$ to a language $\mathscr{L}_2$ including second-order variables and quantifiers. Our semantics should now take care of these, including a domain of possible values of second-order variables. More specifically:

Definition 3 An MV_2*-model* is a structure $\langle D_1, D_2, I\rangle$ such that:

- D_1 is a non-empty domain of quantification.
- D_2 is a set of functions in $\{1, \frac{1}{2}, 0\}^{D_1}$
- $\mathbb{I}$ is an interpretation function identical to that of MV-models except for second-order quantified statements:

  – $\mathbb{I}(\exists X A) = max(\{\mathbb{I}'(A) : \mathbb{I}'$ is an X-variant of $\mathbb{I}\})$
  – $\mathbb{I}(\forall X A) = min(\{\mathbb{I}'(A) : \mathbb{I}'$ is an X-variant of $\mathbb{I}\})$

We might want to impose certain constraints on D_2, like that for each $A \subseteq D_1$ there is an $f \in D_2$ such that $f(a) > 0$ for each $a \in A$. However, we won't force D_2 to contain all functions from D_1 to $\{1, \frac{1}{2}, 2\}$ [8, 408].

Definition 4 We say that $\Gamma \vDash_2^{LP} \Delta$ iff there is no MV_2-model M such that $\mathbb{I}(A) > 0$, for every $A \in \Gamma$ and $\mathbb{I}(B) = 0$ for every $B \in \Delta$.

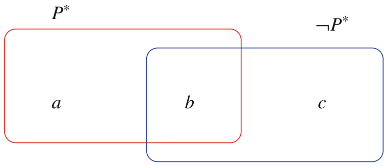
Identity may now be defined in a standard way:

Definition 5 (*Identity*) $(a =_{LP} b) \quad =_{df} \quad \forall P(Pa \equiv Pb)$

1.3 Assessment

Priest's characterization of identity in second-order LP has the effect of "relaxing" some of the properties of classical identity. Consider the following toy model, where for all functions $f \in D_2$, $f(a) = f(b) = f(c)$ except for a function f^* that $f^*(a) = 1$, $f^*(b) = \frac{1}{2}$ and $f^*(c) = 0$. In Priest's dialetheist reading of the semantics, this corresponds to a situation where all the properties are shared similarly by a, b

Table 1 Dialetheist interpretation of $\frac{1}{2}$



and c except for a property P^* that a has (but not its complement), b has (just as its complement) and c lacks (but does have its complement), see Table 1.

N

Identity is both reflexive and symmetric (as it should be). The non-transitivity of identity is inherited from the non-transitivity of LP's material conditional. In the case at hand, for all substitution of P by a predicate interpreted by a function in D2: $Pa \equiv Pb$ and $Pb \equiv Pc$ although it is not the case that for all substitutions of P $(Pa \equiv Pc)$. A second feature is inherited from LP's material conditional. LP's material conditional is not detachable, in the sense that *modus ponens* can fail. This leads, in the case of identity, to a failure of *substitutivity*. The toy model above is a countermodel showing that $b =_{LP} c, \ \ P^*b \not\models_{LP} P^*c$.

Although we find the general approach reasonable, we think the last feature of Priest's proposal is not particularly pleasing. Think of the definition of identity: that is based on the Leibnizian idea according to which identity is a matter of sharing all properties. But the failure of substitutivity clashes with the spirit of the Leibnizian idea. It might be objected that the failure of transitivity is a particular case of failure of substitutivity. That's true, but identity has been defined as sharing all "relevant" properties (note that D_2 need not equal $\{1, \frac{1}{2}, 0\}^{D_1}$). Substitutivity should work at least for "relevant" properties.

In the next section we develop two notions of identity built on ideas close to Priest's. Our first notion of identity is non-transitive but substitutivity works. That's already, we think, an improvement over Priest's notion. Second, we develop a notion of identity that is fully transitive. Despite its classicality, this second notion of identity is sensitive to expressions of (in)definiteness; we want to argue, against the widespread opinion, that a transitive notion of identity is compatible with a metaphysical reading of indefiniteness.

2 Two Notions of Tolerant Identity

2.1 Second-Order ST

In Ripley [9] and Cobreros et al. [2, 3] we investigate a logic that retains some affinities with LP while remaining faithful to classical logic in many respects. The semantics for our logic ST (as we shall call it) is exactly that of LP above. The difference concerns the definition of logical consequence:

Definition 6 We say that $\Gamma \vDash^{ST} \Delta$ iff there is no MV-model M such that $\mathbb{I}(A) = 1$, for every $A \in \Gamma$ and $\mathbb{I}(B) = 0$ for every $B \in \Delta$.

The logic ST sets different standards for satisfaction in premises and in conclusions. A "good" premise (a premise good enough to produce a sound argument) is one that takes value 1. A "good" conclusion, on the other hand (a conclusion that is not false enough to produce a counterexample) is one that takes value greater than 0. This definition might be viewed as setting a *permissive* relation of logical consequence (see [3], Sect. 2.2). For the classical vocabulary (no expressions for indefiniteness or the like) the logic LP coincides with classical logic in its theorems: A is classically valid just in case it is LP-valid. A striking feature of ST is that, for the classical vocabulary, the logic is *fully classical*: Δ is a classical consequence of Γ just in case Δ is an ST-consequence of Γ. However, the logic is sensitive to expressions that do not belong to a purely classical first-order vocabulary (in Cobreros et al. [1] we investigate this logic in connection to the sorites paradox where similarity relations are around; in Cobreros et al. [2] we investigate ST-logic in combination with a transparent truth predicate and self-reference). When non-classical expressions are around, the logic ST might lead to failures of transitivity, thereby blocking inferences that would otherwise trivialize the theory.

The definition of ST carries over from MV to MV_2-models to provide a second order version of this logic:

Definition 7 We say that $\Gamma \vDash_2^{ST} \Delta$ iff there is no MV_2-model M such that $\mathbb{I}(A) = 1$, for every $A \in \Gamma$ and $\mathbb{I}(B) = 0$ for every $B \in \Delta$.

For all that was pointed out above it can be seen that second-order ST is equivalent to (a version of) second-order classical logic.

2.2 Tolerant Identity, First Try

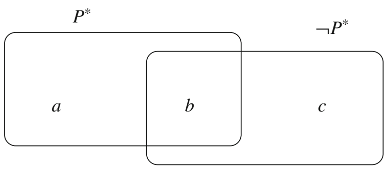
Our first notion of tolerant identity is defined making use of the machinery of MV_2-models above:

Definition 8 ($Tol\,id.\ 1^{st}$). $\mathbb{I}(a \approx b) = 1$ just in case for every $f \in D_2$, $|f(a) - f(b)| < 1$

The expression $|f(a) - f(b)| < 1$ states that a and b are similar with respect to property f. Thus, this definition states that similarity in all properties is sufficient in order to have a corresponding statement of identity good enough to produce a sound argument.

It's easy to see that '$\approx$' is both reflexive and symmetric; and a toy model as the one employed above suffices to show that the relation is not transitive. Recall, that is a model where for all functions $f \in D_2$, $f(a) = f(b) = f(c)$ except for a function f^* that $f^*(a) = 1$, $f^*(b) = \frac{1}{2}$ and $f^*(c) = 0$. The ST-reading of the semantics is different from the LP reading, though. Now values are relative to the position of

Table 2 ST interpretation of $\frac{1}{2}$



corresponding sentences in premises or conclusions of an argument. In Table 2, the upper oval indicates a "good conclusion" (cannot produce a counterexample) and lower oval a "bad premise" (cannot produce a sound argument).

This notion of identity does retain substitutivity in the sense that the following properties hold:

(Subst1) $\models_2^{ST} \forall x \forall y \forall P((Px \wedge x \approx y) \supset Py)$
(Subst2) $Px, x \approx y \models_2^{ST} Py$

Note that in order for (Subst1) to fail, the conditional should take value 0. This occurs just in case the antecedent is 1 and the consequent is 0. But if '$Px \wedge x \approx y$' takes value 1, then Py should take at least value $\frac{1}{2}$. Similarly for (Subst2): for that inference to fail there must be a model where premises are 1 and conclusion is 0. But the value 1 of '$Px, x \approx y$' guarantees a value greater than 0 for Py.

2.3 Tolerant Identity, Second Try

Our second definition of identity directly mirrors Priest's strategy but within the (second-order) ST-logic.

Definition 9 ($Tol\,id.2^{nd}$). $(a = b) \quad =_{df} \quad \forall P(Pa \equiv Pb)$

Despite the affinities in the semantics, the classicality of ST's material conditional makes this notion of identity fully transitive. This notion of identity is, however, sensitive to non-classical expressions. Consider Priest's motorbike once again. At each stage, the resulting motorbike is similar in all its properties to the previous one. That is, for each of the stages a_n of the motorbike we have $a_n \approx a_{n+1}$ ('$\approx$' understood as defined in the previous section). Although the notion of identity introduced in Definition 9 is classical, it is tolerant in connection to the similarity relation introduced in Definition 8. That is, the following *tolerance principles for identity* hold in ST:

TPI1 $\models_2^{ST} \forall x \forall y((PM = x \wedge x \approx y) \supset PM = y)$
TPI2 $a \approx b,\ a = PM \models_2^{ST} b = PM$

Table 3 Evans' argument

1	$\nabla(b = a)$	(Assumption)
2	$\lambda x[\nabla(x = a)]b$	(From 1, by abstraction)
3	$\neg\nabla(a = a)$	(Assumption, since $a = a$ is a logical truth!)
4	$\neg\lambda x[\nabla(x = a)]a$	(From 3, by abstraction)
5	$\neg(a = b)$	(From 2, 4, by LL and Contrap)
6	$\neg\nabla(a = b)$	(Assuming premises are definite)

In short, although identity is fully classical, it is a tolerant relation (unlike standard classical identity). This fact can be used to explain our intuitions about non-sharp transitions in cases like that of Priest's motorbike. At the same time, the non-transitivity of the ST-logic is what prevents the unwelcome conclusion of the sorites paradox.

3 Transitivity and Metaphysical Indeterminacy

In the previous section we argued that Priest's approach to identity can be refined. First, the failure of substitutivity deprives the Leibnizian definition of identity of its intended force. Within the ST-logic, we can define a non-transitive notion of identity for which substitutivity works. Second, within the ST-logic, we can define a notion of identity that is fully transitive (and, naturally, for which substitutivity works) but that is tolerant, and so it makes still room for indeterminacy. In this section we want to argue that the indeterminacy associated to this notion of identity need not be understood in a purely semantic way. Thus, against a widespread opinion, we argue that transitivity of identity and metaphysical indeterminacy are compatible.

In order to show this, we consider Evans' famous argument (in [4]), under Lewis' interpretation (in [7]). Evans' argument is a *reductio* from the assumption of a true statement of indefiniteness of identity ('∇A' means 'it is indefinite whether A'). See Evans' arguement in Table 3.

Naturally, this argument must be fallacious, since it is perfectly agreed that there might be indefinite identity statements. Lewis' interpretation of Evans' argument is that while the defender of indeterminacy as semantic can easily point out where the fallacy lies, the same is not the case for the defender of indeterminacy as metaphysical. In particular, one can say that the steps from 1 to 2 and 3 to 4 are not valid, in much the same way as the inference from the true statement,

'It is contingent whether the number of planets is eight'

does not entail the false statement,

'the number of planets is such that it is contingent whether it is eight'.

Now this analogy makes perfect sense if indeterminacy is understood in terms of a variation of the denotation of a term across precisifications (that is the supervaluationist reading). In that case, 'contingency' and 'indeterminacy' are formally identical and the mentioned inference is not valid. But for the defender of indeterminacy as metaphysical, indeterminacy cannot be explained in terms of variation of the denotation across anything. The terms a and b in the argument "rigidly denote" (to follow the modal analogy) an object that is intrinsically vague.

We take Evans' argument (under Lewis' interpretation) as a criterion for the availability of a metaphysical reading of indeterminacy associated to identity. Given notions of identity and of indeterminacy, if the only way to block the argument is the invalidity of abstraction into the scope of the ∇ operator, then that notion of indeterminacy has just a semantic reading.

Let now definiteness ('it is definite *that*') be defined as follows:

Definition 10 (*Definiteness*)

$$\mathbb{I}(\mathsf{D}(a=b)) = \begin{cases} 1 \text{ if for all } f \in D_2 \quad |f(a) - f(b)| = 0 \\ 0 \text{ otherwise} \end{cases}$$

Indefiniteness ('it is indefinite *whether*') as expressed by '∇' can be defined thus:

$$\nabla(a=b) = \neg\mathsf{D}(a=b) \wedge \neg\mathsf{D}\neg(a=b),$$

and consider again Evans' argument. Each step in the argument is ST-valid. However, (5) is only tolerantly true and (6) is not even tolerantly true. Thus, though each step is valid we cannot validly chain premises in this case.

4 Conclusion and Outlook

Priest argued that to account for substantial change, one must admit that identity is non-transitive. Making use of the logic LP, he defines such a notion of identity, which also fails to satisfy substitutivity. Making use of our alternative logic ST, we have shown that we can not only define a non-transitive notion of identity that preserves substitutivity, but also a notion of identity that is transitive. The latter is particularly interesting, because even though transitive, it is still a tolerant relation which allows for substantial change. Addressing Evans' argument, we have also shown that the transitivity of identity is compatible with ontological vagueness.

In this chapter we have focussed on logical issues. However, we believe that our proposed analyses of identity have interesting ontological implications. We mentioned already the issues of substantial change and ontological vagueness. But both deserve more extensive discussion: how is substantial change compatible with the transitivity of identity from a conceptual point of view, and what does it mean to be a vague object? Identity is crucial for counting, but what is the consequence for counting when our notions of identity are used? Last but not least, there is Geach's

[5] notion of 'relative identity' and Unger's [10] problem of the Many. We feel that for both a fresh perspective becomes available when use is made of the notions introduced in this chapter. We hope to address these issues in a subsequent chapter.

References

1. Cobreros, P., Egre, P., Ripley, D., & van Rooij, R. (2012). Tolerant, classical, strict. *Journal of Philosophical Logic*, *41*(2), 347–85.
2. Cobreros, P., Egré, P., Ripley, D., & van Rooij, R. (2014). Reaching transparent truth. *Mind* (forthcoming).
3. Cobreros, P., Egré, P., Ripley, D., & van Rooij, R. (2014). Vagueness, truth and permissive consequence. In T. Achourioti, H. Galinon, K. Fujimoto & J. Martínez-Fernández (Eds.), *Unifying the Philosophy of Truth*. Springer (forthcoming).
4. Evans, G. (1978). Can there be vague objects? *Analysis*, *38*(4), 208.
5. Geach, P. T. (1980). *Reference and generality* (3rd ed.). Ithaca: Cornell University Press.
6. Lewis, D. (1986). *On the plurality of worlds*. Oxford: Blackwell.
7. Lewis, D. (1988). Vague identity: Evans misunderstood. *Analysis*, *48*(3), 128–30.
8. Priest, G. (2010). Non-transitive identity. In R. Dietz & S. Moruzzi (Eds.), *Cuts and Clouds*. Oxford: Oxford University Press.
9. Ripley, D. (2012). Conservatively extending classical logic with transparent truth. *The Review of Symbolic Logic*, *5*(2), 354–78.
10. Unger, P. (1980). The problem of the many. *Midwest Studies in Philosophy*, *5*(1), 411–67.