J Philos Logic (2015) 44:771–780
DOI 10.1007/s10992-015-9358-6

Paraconsistent Logic

David Ripley

Received: 31 January 2013 / Accepted: 15 June 2013 / Published online: 27 February 2015

Abstract In some logics, anything whatsoever follows from a contradiction; call these logics *explosive*. *Paraconsistent logics* are logics that are not explosive. Paraconsistent logics have a long and fruitful history, and no doubt a long and fruitful future. To give some sense of the situation, I'll spend Section 1 exploring exactly what it takes for a logic to be paraconsistent. It will emerge that there is considerable open texture to the idea. In Section 2, I'll give some examples of techniques for developing paraconsistent logics. In Section 3, I'll discuss what seem to me to be some promising applications of certain paraconsistent logics. In fact, however, I don't think there's all that much to the concept 'paraconsistent' itself; the collection of paraconsistent logics is far too heterogenous to be very productively dealt with under a single label. Perhaps that will emerge as we go.

Keywords Paraconsistent logic · Negation · Explosion

1 What is Paraconsistency?

There are at least two notions of paraconsistency already in the literature. They are sometimes referred to as 'strong paraconsistency' and 'weak paraconsistency' (eg in [4, p. 126], [20, p. 96], [14, p. 158]), but this terminology has the drawback that there is nothing particularly stronger or weaker about one than the other. They

D. Ripley (✉)
University of Connecticut, Mansfield, CT USA
e-mail: davewripley@gmail.com

are simply independent properties of a consequence relation.[1] I'll refer to them, then, with the more descriptive names 'conjunctive paraconsistency' and 'collective paraconsistency'. Here is a first pass at these notions:

Definition 1 (Conjunctive paraconsistency) A consequence relation $\vdash$ on a language $\mathcal{L}$ is *conjunctively paraconsistent* iff there are wffs $A, B \in \mathcal{L}$ such that $A \wedge \neg A \nvdash B$.

Definition 2 (Collective paraconsistency) A consequence relation $\vdash$ on a language $\mathcal{L}$ is *collectively paraconsistent* iff there are wffs $A, B \in \mathcal{L}$ such that $A, \neg A \nvdash B$.

In a logic for which $A \wedge B \vdash C$ implies $A, B \vdash C$, collection-paraconsistency implies conjunction-paraconsistency; in a logic for which the reverse is true, the reverse is true. (The terminology of 'strong' and 'weak' paraconsistency thus assumes one direction of this implication but not the other.)

These definitions can only be a first pass, however, as both depend on some prior understanding of $\neg$ (meant to be a negation), or at least of which wff $\neg A$ is for a given A. In addition, conjunctive paraconsistency depends as well on a similar prior understanding of $\wedge$ (meant to be a conjunction), and collective paraconsistency on a prior understanding of the structural comma occurring between premises, of what it is to take premises together.

In some restricted logical settings, all this can be taken for granted: there is exactly one plausible candidate to fill each role. In many other settings, though, it cannot. For example, the logic $\mathrm{CR}^{\star}$ of [23] has two negations. One behaves (conjunction-)paraconsistently; the other does not. On the other hand, affine logic (linear logic plus weakening; see eg [36] for details) has one negation, but two conjunctions. Again, one behaves paraconsistently, while the other does not.[2] Still other systems, like the display systems of [8], feature multiple ways for premises to be combined, such that whether a logic counts as collection-paraconsistent can depend on what kind of collection is in play. Some conjunctions and ways of combining premises are order-sensitive, so it can matter whether we consider A together with $\neg A$ or $\neg A$ together with A. And there are systems, like some of those of [17], that exhibit all of these features. For such systems, the question 'Is it paraconsistent?' is hopelessly imprecise.

The only answerable questions, then, will be more specific, selecting at least a particular negation together with either a particular conjunction or a particular way of combining premises.[3] Worse, this selection must happen *after* the logical system

[1] I have nothing much to say about which things are and which things are not consequence relations. I certainly have no precise definition in mind; usual precise definitions exclude some things I include. (For example, understanding a consequence relation as a Tarskian closure operation excludes a wide variety of substructural logics.) If it so much as smells consequencey it's a consequence relation as far as I'm concerned.

[2] For the additive conjunction $\wedge$, we have $A \wedge \neg A, A \wedge \neg A \vdash B$, but $A \wedge \neg A \nvdash B$; two copies of a contradiction do entail everything, but a single copy does not.

[3] You might note that I haven't said anything about what it takes to count as a negation or a conjunction or a way of combining premises in the first place. No way am I going near *that* can of worms.

in question is specified, so we can see just which connectives occur to be asked after; there's no sense asking about the paraconsistency with regard to ¬ and & of a logic with no &. As a result, precise notions of paraconsistency don't travel well; it can be very difficult to ask whether two distinct logics *share* a particular property of paraconsistency, or whether they exhibit different kinds, since we'd need some way to identify connectives across logics.

But for the sake of being able to talk more easily about paraconsistency, let's pretend these difficulties don't arise. For the remainder of the paper, I'll focus on logics in which there is a single best candidate for each of the roles of negation, conjunction, and premise-collector; as a result, we will have only the two properties of conjunction-paraconsistency and collection-paraconsistency to worry about. Indeed, in many of these logics it will be the case that $A, B \vdash C$ iff $A \wedge B \vdash C$, and for these I'll talk simply of 'paraconsistency'.

Even with these assumptions in place, it can be tempting to refine the notion further. For example, minimal logic, while technically paraconsistent, is such that $A \wedge \neg A \vdash \neg B$ for every A, B. But it seems that whatever reasons someone might have for paraconsistency are likely to also be reasons to avoid this kind of situation. As a result, there might be reason to develop a new notion that would both entail paraconsistency and rule out minimal logic, along with such other critters as might exhibit relevantly similar behaviour. ([45] makes a reasonably compelling attempt at the task.) I won't pursue this here; I'll just note that minimal logic and the like obey the letter of paraconsistency without quite getting the spirit right.

2 How to be Paraconsistent

In this section, I discuss some ways in which paraconsistent systems can be constructed. I'll focus on model-theoretic presentations here, since paraconsistency is a *non*entailment claim. Models at their most general are wildly varied critters. In presenting a particular logic model-theoretically, if even a single counterexample to a single instance of explosion gets in, the logic is paraconsistent. But there are many ways for something to count as a counterexample to an instance of explosion. As a result, it is easy to specify paraconsistent logics model-theoretically; indeed, one has to go to special trouble to specify an explosive logic, ruling out every possible kind of counterexample. So there are lots of options here.

2.1 Designation

Suppose we're working with model theories in which consequence is preservation of some status. In such model theories, there is some set of *designated* wffs in each model; a model is a counterexample to an argument iff it designates all the premises of the argument and none of its conclusions. In order to present a paraconsistent logic in this way, there must be models on which $A \wedge \neg A$ is designated, or on which both A and $\neg A$ are designated.

Here, I'll give a few examples of ways in which this can be done for various paraconsistent logics, demonstrating some of the techniques that can be used. Note at the outset, though, that there is no reason to expect any difficulty in achieving paraconsistency; the only way to stop $A \wedge \neg A$ from being designated is to impose some restriction that rules it out. Don't impose that restriction, and you're good to go. The real import of the techniques to follow is the way in which they allow for $\wedge$ and $\neg$ to be recognizably conjunctiony and negationy, respectively, without making the logic explosive.

One way to find a countermodel to explosion is to build models that split A off from $\neg A$, evaluating each formula with respect to a different setting of some parameter. For example, one way to model subvaluationist logic is via is a pile of classical models that share a domain; it is a model of a set of sentences iff for every sentence in the set, some classical model in the pile is a model of the sentence.[4] The set $\{A, \neg A\}$ is easy to model in this way: simply pile a classical model of A together with one of $\neg A$. As it is possible to do this without including any classical models of B in the pile, for at least some B, it follows that subvaluational consequence is collection-paraconsistent.

Subvaluationism can only split A off from $\neg A$ in this way if they are not conjoined, as it works sentence-by-sentence. As a result, it is not conjunction-paraconsistent, since no classical model is a model of $A \wedge \neg A$. But by shifting the truth conditions for conjunction, we can achieve conjunction-paraconsistency via a similar strategy, by allowing designation, once achieved, to flow from conjuncts to the conjunction. This sort of approach to conjunction-paraconsistency is explored in [34], yielding the logic LP, which is both collection- and conjunction-paraconsistent.[5]

Another well-trodden route to paraconsistency is *indeterminism*. For example, a model theory for the $\neg$-$\wedge$-$\vee$ fragment of Da Costa's system C_ω is given in [32, p. 162]. Models, on this approach, assign either 1 or 0 to each formula of the language; the values assigned to conjunctions and disjunctions are determined by the values assigned to their conjuncts and disjuncts in the expected way. But the values assigned to negations are not determined by the values assigned to their negata, only constrained. In particular, A and $\neg A$ cannot both receive the value 0, and A must receive the value 1 if $\neg\neg A$ does. (The logic PI given in [5, p. 191], whose predicate extension has become known as CLuN [6], drops the very last requirement and is otherwise the same.) This leaves open the possibility for both A and $\neg A$ to take value 1, even while some B take value 0, so this approach too allows for counterexamples to explosion.[6]

It's also common to give designation-based model theories for paraconsistent logics that rely neither on relativization nor on indeterminism. For example, the usual three-valued model-theoretic presentation of the logic LP simply adds a third value to the usual classical pair of values, and extends the functions that interpret the classical connectives to functions on the enriched value space. In particular, negation takes the new value to itself. Both the old classical 1 and the new value are designated; as a result, if a wff takes the new value, both it and its negation are designated.

[4]This works for the subvaluationist 'global' consequence. For more on subvaluationist logic, see [11, 19, 46, 47].

[5]For more on LP, see [7, 26, 29].

[6]For more on the C systems, see [10, 13].

This allows for counterexamples to explosion without any kind of relativization or indeterminism.[7]

We can recast the relativized approach in this direct form. For example, we can give an unrelativized model theory for subvaluationist logic by using Boolean algebras; say that a formula is designated on an interpretation into such an algebra iff its interpretation is not the bottom element of the algebra. As it's possible for neither A nor $\neg A$ to be interpreted as the bottom, the collection-paraconsistency of subvaluationist logic is respected. Since it's not possible for $A \wedge \neg A$ to be interpreted as anything other than the bottom, the conjunction-explosiveness of subvaluationist logic is respected too.

The so-called 'Australian plan' for models for relevant logics uses a variant of the relativized approach. The models in question consist of a number of points at which formulas are evaluated, and each point a has an associated point $a^\star$ (this star is the so-called 'Routley star', named for [41]). This is a direct approach, since both A and $\neg A$ can hold at a single point; but it shares something with relativized approaches, since $\neg A$ holds at a point a iff A does not hold at the possibly distinct $a^\star$. (For more on the Routley star, see [15, 35].)

One nice feature of this approach is that it allows for structure to be passed between positive and negative formulas. That is, constraints on when positive sentences can or cannot be designated turn out to automatically influence when negative sentences can or cannot be designated. For example, if conjunction exhibits familiar behaviour at a point $a^\star$—that $A \wedge B$ holds there iff both A and B do—then we directly have corresponding behaviour for negated conjunctions at a—that $\neg(A \wedge B)$ holds there iff either $\neg A$ or $\neg B$ does. If the conjunction constraint holds for all points (as it often does), then the negated-conjunction constraint too holds for all points. For simple connectives like conjunction and disjunction, this is only a little simplification, but when more involved connectives like relevant conditionals are added to the language the simplification becomes very welcome indeed, and is one of the main reasons why the 'Australian plan' is in more common use than the 'American plan', which does not share this feature. (For more on the difference, see [40].)

2.2 Doing Without Designation

Failure to preserve a particular status is only one option, however, for saying when a model is a counterexample to an argument. As a result, there are ways to give counterexamples to explosion in certain logics without having to assign a special positive status to any contradictions.

For example, many logics admit of a model theory on which a counterexample is an interpretation in a complete lattice of an appropriate sort (which sort depends on the logic in question) such that the greatest lower bound of the interpretations of the premises does not stand in the lattice's ordering relation to the least upper bound of the interpretations of the conclusions. An example of this approach is the De Morgan

[7] Note that there is still something of an indeterministic flavour: if you know that A is designated, that's not yet enough to say whether $\neg A$ is designated or not. But the facts that underlie this are fully deterministic.

algebraic model theory given by Dunn [3, p. 180–206] for the paraconsistent logic FDE.

By allowing for variations that replace greatest lower bound and least upper bound in this approach with other sorts of operations, we can capture even more logics. This way of formulating a logic, though, makes no attempt to see the logic as preserving any particular status.[8] On approaches like this, paraconsistency is likewise easy to achieve. It would take work to make sure that $A \wedge \neg A$ is always interpreted below B, for any A and B. (The easiest way is to ensure that $A \wedge \neg A$ is always interpreted as a minimum element.) By not bothering to do that work, paraconsistency is achieved.

There are other approaches as well. For example, [21] presents q-consequence operations by using two distinct notions of designation: on this approach, a counterexample to an argument takes all its premises to a value that is designated in one sense, but takes its conclusion to a value that is not designated in the other sense. This kind of approach can be used to specify a range of consequence relations that do not fit into other approaches, and many of them are paraconsistent. (Indeed, Malinowksi's strategies enable us to consider a range of consequence relations that extends all the way down to the empty relation—and this is certainly paraconsistent!)

3 Applications of Paraconsistent Logics

This section will sketch some applications of paraconsistent logics. It's a biased sketch, pointing mainly to issues I can say a little bit about; in no way does it come close to exhausting the available applications. These applications are of particular logics or particular kinds of logics; it is certainly not the case that just any paraconsistent logic will do. But it might help to give a sense of the sort of research programs that paraconsistent logics naturally fit into. These applications have a predictable thing in common: they involve collections of sentences containing some contradictions (either $A \wedge \neg A$ or the pair A, $\neg A$), together with some desire to draw reasonable conclusions from the set.

3.1 Compositional Semantics

Paraconsistent logics have arisen in compositional semantic theories of natural languages in two main ways: first, as auxiliaries to theories of clause-embedding environments of various sorts; and second, in the study of vague predicates.

3.1.1 Embedding Clauses

A compositional semantics for natural languages must eventually grapple with questions about the relations between clause-embedding environments and negation. It is

[8] Any fully structural logic can be seen as preserving *some* status; this is one upshot of Suszko's Thesis; see eg [16, 18, 43, 44]. But this is sometimes not the most helpful way to look at it. Moreover, the suggested generalization of this method extends well beyond fully structural logics, as in [22].

a commonplace that we all have some inconsistent beliefs.[9] Yet we do not all believe everything. So there are cases where ‘x believes that A’ and ‘x believes that $\neg A$’ are both true, but ‘x believes that B’ is not. The *internal logic* of belief, then, is paraconsistent. The same point can be made for a wide variety of clause-embedding constructions.

This does not show that our language is governed by a paraconsistent logic (whatever that might mean), but it is enough to reveal how paraconsistent logics, and techniques developed by paraconsistent logicians, can be of use in studying natural language, whether the language as a whole is governed by a paraconsistent logic or not. The point is a familiar one, made for example in [42], but it has been revived in recent years, and put to use in eg [28, 39].

3.1.2 Vagueness

A comprehensive theory of natural language must also say something about vague predicates and their entailments. As explained in [19], there has long been an idea that paraconsistency—and indeed *in*consistency—ought to play some role here. This is because one of the key features of vague predicates is their having *borderline cases*, and one natural way to say that something is a borderline case of, say, blue, is to say that it’s both blue and not blue.

Whether people who say things like this ‘really mean’ to contradict themselves or not, paraconsistent logics provide useful tools for modeling and understanding this way of talking. For example, there is some evidence that speakers agree to ‘both P and not P’ in the same kinds of circumstances in which they agree to ‘neither P nor not P’, for vague predicates P.[10] A number of paraconsistent logics exhibit this same feature; it is, after all, a consequence of a De Morgan equivalence together with a double negation equivalence.

There is of course some controversy as to how best to model the evidence currently available, and there is still much we do not know. For further discussion that shows in more detail how paraconsistent logics of various stripes can be useful here, see [1, 12, 38, 51].

3.2 Inconsistent Mathematics

Mathematics involves the precise development of theories about a wide range of things. Some of these theories are couched in classical logic, and some are not. To put the point this way is perhaps contentious; the *vast* majority of contemporary mathematics is conducted under the assumption of classical logic. But there have long been exceptions.

[9]Typically, this is because we haven’t noticed the contradiction, but there are at least two other kinds of case: 1) cases in which we notice the contradiction, but haven’t yet decided how or whether to resolve it, and 2) cases in which we notice the contradiction, but have decided simply to live with it.
[10]See [2, 37] for details.

Constructive mathematics provides perhaps the best-known kind of exception. Constructive mathematics is not simply the study of familiar classical mathematical objects via a weaker constructive logic, like looking at something with one eye closed to see how good your other eye is on its own. Rather, adopting a constructive logical framework allows for the study of mathematical objects that cannot exist in a classical framework. Weaken the logic, and you can strengthen the axioms without becoming overstrong.[11]

Paraconsistent logics allow for axioms to be strengthened past the point of inconsistency without yet becoming overstrong. This is useful for exploring a variety of mathematical entities that cannot be seen through a classical lens owing to paradoxes of various sorts. Within set theory, for example, paraconsistent approaches allow for the study of such critters as the russell set, the set of all ordinals, etc. Moreover, these sets can be seen to have the (contradictory) properties they ‘should have’: in particular, both the russell set and the set of all ordinals both are and are not members of themselves. In a paraconsistent setting, this can be handled without setting off the explosions that would result elsewhere.

In a variety of different logical and axiomatic settings, of course, a variety of results are forthcoming. Here, as elsewhere in mathematics, it is better for ten thousand flowers to bloom. (See in this connection [30], as well as the full special issue in which that paper appears.) There is a strong tradition of paraconsistent mathematics (for examples and further references, see eg [9, 24, 27, 33]) that continues to thrive today, being carried forward in a younger generation by thinkers such as Weber in the Australian relevant tradition (eg in [49, 50]) and Verdée in the Belgian adaptive tradition (eg in [48]).

4 Conclusion

To sum: there are a lot of different properties one might have in mind by ‘paraconsistent’, there are a lot of different techniques one can use to construct counterexamples to explosion, and paraconsistent logics find application in a wide variety of projects, ranging from the study of natural language to pure mathematics, and well beyond. There is also a fascinating history of interaction between various approaches and schools, which I haven’t had space (or expertise) to go into here. I guess it might be the history that explains why ‘paraconsistent logic’ can sometimes seem like one topic. For an outline, see perhaps [4, 31].

References

1. Alxatib, S., Pagin, P., & Sauerland, U. (2013). Acceptable contradictions: Pragmatics or semantics? a reply to Cobreros et al. *Journal of Philosophical Logic*, *42*(4), 619–634.

[11] Smooth infinitesimal analysis provides an example. It is a rich topic of mathematical study based on axioms that are inconsistent in classical logic, but not in the intuitionist logic in which it is conducted. See eg [25].

2. Alxatib, S., & Pelletier, J. (2011). The psychology of vagueness: Borderline cases and contradictions. *Mind and Language*, *26*(3), 287–326.
3. Anderson, A.R., & Belnap, N.D. (1975). *Entailment: The Logic of Relevance and Necessity, vol. 1*. Princeton: Princeton University Press.
4. Arruda, A.I. (1989). Aspects of the historical development of paraconsistent logic. In [33] (pp. 99–130).
5. Batens, D. (1989). Dynamic dialectical logics. In [33] (pp. 187–217).
6. Batens, D. (2000). Towards the unification of inconsistency handling mechanisms. *Logic and Logical Philosophy*, *8*, 5–31.
7. Beall, J., & van Fraassen, B.C. (2003). *Possibilities and Paradox: An Introduction to Modal and Many-valued Logic*. Oxford: Oxford University Press.
8. Belnap, N.D. (1982). Display logic. *Journal of Philosophical Logic*, *11*(4), 375–417.
9. Brady, R.T. (2006). *Universal Logic*. Stanford: CSLI Publications.
10. Carnielli, W.A., & Marcos, J. (1999). Limits for paraconsistent calculi. *Notre Dame Journal of Formal Logic*, *40*(3), 375–390.
11. Cobreros, P. (2013). Vagueness: Subvaluationism. *Philosophy Compass*, *8*(5), 472–485.
12. Cobreros, P., Égré, P., Ripley, D., & van Rooij, R. (2012). Tolerant, classical, strict. *Journal of Philosophical Logic*, *41*(2), 347–385.
13. da Costa, N. (1974). On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, *15*, 497–510.
14. Dietz, R. (2011). The paradox of vagueness. In Horsten, L., & Pettigrew, R. (Eds.) *The Continuum Companion to Philosophical Logic*, (pp. 128–179). Great Britain: Continuum.
15. Dunn, J.M. (1993). Star and perp: Two treatments of negation. *Philosophical Perspectives*, *7*, 331–357.
16. Dunn, J.M., & Hardegree, G.M. (2001). *Algebraic Methods in Philosophical Logic*. Oxford: Oxford University Press.
17. Goré, R. (1998). Gaggles, Gentzen, and Galois: How to display your favorite substructural logic. *Logic Journal of the IGPL*, *6*(5), 669–694.
18. Humberstone, L. (2012). *The Connectives*. Cambridge: MIT Press.
19. Hyde, D. (1997). From heaps and gaps to heaps of gluts. *Mind*, *106*, 641–660.
20. Hyde, D. (2008). *Vagueness, Logic, and Ontology*. Hampshire: Ashgate.
21. Malinowski, G. (1990). Q-consequence operation. *Reports on Mathematical Logic*, *24*, 49–59.
22. Meyer, R.K., & Routley, R. (1972). Algebraic analysis of entailment I. *Logique et Analyse*, *15*, 407–428.
23. Meyer, R.K., & Routley, R. (1974). Classical relevant logics II. *Studia Logica*, *33*(2), 183–194.
24. Mortensen, C. (1995). *Inconsistent Mathematics*. Dordrecht: Kluwer Academic Publishers.
25. O'Connor, M. (2008). An introduction to smooth infinitesimal analysis. arXiv:0805.3307 [math.GM].
26. Priest, G. (1979). Logic of paradox. *Journal of Philosophical Logic*, *8*, 219–241.
27. Priest, G. (2002). Paraconsistent logic. In Gabbay, D.M., & Guenthner, F. (Eds.) *Handbook of Philosophical Logic*, 2nd edn. Vol. 6. Dordrecht: Kluwer Academic Publishers. 2 edition.
28. Priest, G. (2005). *Towards Non-Being: The Logic and Metaphysics of Intentionality*. Oxford: Oxford University Press.
29. Priest, G. (2008). *An Introduction to Non-Classical Logic: From If to Is*, 2nd edn. Cambridge: Cambridge University Press.
30. Priest, G. (2013). Mathematical pluralism. *Logic Journal of the IGPL*, *21*(1), 4–13.
31. Priest, G., & Routley, R. (1989a). First historical introduction: A preliminary history of paraconsistent and dialetheic approaches. In [33] (pp. 3–75).
32. Priest, G., & Routley, R. (1989b). Systems of paraconsistent logic. In [33] (pp. 151–186).
33. Priest, G., Routley, R., & Norman, J. (Eds.) (1989). *Paraconsistent Logic: Essays on the Inconsistent*. Munich: Philosophia Verlag.
34. Restall, G. (1997). Ways things can't be. *Notre Dame Journal of Formal Logic*, *38*(4), 583–596.
35. Restall, G. (1999). Negation in relevant logics: How I stopped worrying and learned to love the Routley star. In Gabbay, D., & Wansing, H. (Eds.) *What is Negation?* (pp. 53–76). Dordrecht: Kluwer Academic Publishers.
36. Restall, G. (2000). *An Introduction to Substructural Logics*. London: Routledge.
37. Ripley, D., Nouwen, R., Rooij, R., Schmitz, H.-C., & Sauerland, U. (2011a). Contradictions at the borders. *Vagueness and Communication*, (pp. 169–188): Springer.

38. Ripley, D. (2011b). Inconstancy and inconsistency. In Cintula, P., Fermüller, C., Godo, L., & Hájek, P. (Eds.) *Understanding Vagueness: Logical, Philosophical, and Linguistic Perspectives*, (pp. 41–58): College Publications.
39. Ripley, D. (2012). Structures and circumstances: Two ways to fine-grain propositions. *Synthese*, *189*(1), 97–118.
40. Routley, R., Meyer, R.K., Plumwood, V., & Brady, R.T. (1982). *Relevant Logics and their Rivals 1*. Atascadero: Ridgeview.
41. Routley, R., & Routley, V. (1972). The semantics of first degree entailment. *Noûs*, *6*(4), 335–359.
42. Routley, R., & Routley, V. (1975). The role of inconsistent and incomplete theories in the semantics of belief. *Communication and Cognition*, *8*, 185–235.
43. Shoesmith, D.J., & Smiley, T.J. (1978). *Multiple-conclusion Logic*. Cambridge: Cambridge University Press.
44. Shramko, Y., & Wansing, H. (2012). Truth values. In Zalta, E.N. (Ed.) *The Stanford Encyclopedia of Philosophy*. Winter 2012 edition.
45. Urbas, I. (1990). Paraconsistency. *Studies in Soviet Thought*, *39*(3/4), 343–354.
46. Varzi, A. (1997). Inconsistency without contradiction. *Notre Dame Journal of Formal Logic*, *38*(4), 621–639.
47. Varzi, A. (2000). Supervaluationism and paraconsistency. In Batens, D., Mortensen, C., Priest, G., & Bendegem, J.-P.V. (Eds.) *Frontiers in Paraconsistent Logic*, (pp. 279–297). Baldock: Research Studies Press.
48. Verdée, P. (2013). Strong, universal, and provably non-trivial set theory by means of adaptive logic. *Logic Journal of the IGPL*, *21*(1), 108–125.
49. Weber, Z. (2010). Transfinite numbers in paraconsistent set theory. *Review of Symbolic Logic*, *3*(1), 71–92.
50. Weber, Z. (2012). Transfinite cardinals in paraconsistent set theory. *Review of Symbolic Logic*, *5*(2), 269–293.
51. Weber, Z., Ripley, D., Priest, G., Hyde, D., & Colyvan, M. (2014). Tolerating gluts: a reply to Beall. *Mind*, *123*(491), 813–828.