Why I am not a noncontractivist

David Ripley

University of Connecticut

SILFS satellite 2014, Rome

Why I am a substructuralist

Triple-N paradoxes

Triple-N paradoxes come in many forms:

What's naive:

truth, satisfaction, reference, validity, membership

What's negative:

negations, conditionals, generalized quantifiers, validity

What's neverending:

self-reference, reference loops, infinite chains

But they form a family:
there is some single phenomenon here.

Validity curry and Yablo paradox have something in common,
in contrast with Zeno paradoxes
or sorites paradoxes
or the paradoxes of material implication.

(The inclosure schema is miscalibrated:
it misses Curries, and includes sorites.)

Why I am a substructuralist

Uniform solution

PUS (Priest 1994)

‘*[S]ame kind of paradox, same kind of solution*’

How to address the NNNs?

Not by attention to:

truth, negation, self-reference, conditionals, validity, membership, etc.

Each of these is inessential!

Take a NNN paradox, and reason your way to triviality.

What have you appealed to?

Take a NNN paradox, and reason your way to triviality.

What have you appealed to?

Two safe bets: contraction, and cut.

These two are at the scene of every crime;
they should be very high on our list of suspects.

Nothing else turns up so generally.

Contraction:

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}$$

Cut:

$$\frac{\Gamma' \vdash A \qquad \Gamma, A \vdash B}{\Gamma, \Gamma' \vdash B}$$

From transitivity to contraction

Why transitivity?

The problem for noncontractivists:

any good motivation for accepting transitivity
turns out to push for contraction too.

Two good motivations for transitivity:

— The argument from lemmas —
— The argument from closure —

From transitivity to contraction

The argument from lemmas

Ordinary reasoning involves establishing lemmas:
subsidiary conclusions that we draw on in further reasoning.

But this seems to require transitivity.

But how does ordinary reasoning draw on lemmas?
By way of cumulative reasoning.

Cumulative reasoning from a body of information:

1. Draw conclusions validly from the info you have.
2. Add those conclusions to the info you have.
3. Repeat ad lib.
4. Any eventual conclusion has been reached from the starting point.

Cumulative reasoning requires cautious cut.

Cautious cut:

$$\frac{\Gamma \vdash A \qquad \Gamma, A \vdash B}{\Gamma \vdash B}$$

This sure looks like cut—but it ain't.

Cumulative reasoning requires cautious cut.

Cautious cut:

$$\frac{\Gamma \vdash A \qquad \Gamma, A \vdash B}{\Gamma \vdash B}$$

Consider:

$$\frac{\overline{\Gamma, A \vdash A} \qquad \Gamma, A, A \vdash B}{\Gamma, A \vdash B}$$

From transitivity to contraction

The argument from closure

We can present a big theory via a small part,
so long as the theory is the closure of the small part.

But closures seem to require transitivity.

A closure operation C on a poset is an operation such that:

Closure conditions:

(inc) $x \leq Cx$

(mon) $x \leq y \Rightarrow Cx \leq Cy$

(idem) $Cx = CCx$

These are all needed to play the closure role!

Nothing prevents them applying to multisets,
ordered by submultiset ($\sqsubseteq$).

Let $X \sqcup Y$ be multiset union understood as maximum (not sum!).

Fact:

For any closure C on multisets (ordered by $\sqsubseteq$),
if $A \in C(X)$ and $B \in C(Y \sqcup [A])$,
then $B \in C(Y \sqcup X)$.

This sure looks like cut—but it ain't.
Cut needs $\uplus$: union as sum.

Nothing like a familiar noncontractive consequence relation can be understood as a closure on multisets.

Proof:

- Take:
  - formulas A, B, D and multiset $X \not\ni B$ such that:
  - $X, A, A \vdash D$ but $X, A \nvdash D$, and
  - A and B are distinct but entail each other.
- Suppose a closure C with $E \in C(Y)$ iff $Y \vdash E$.
- Since $X, A, A \vdash D$ and $B \vdash A$, by cut $X, A, B \vdash D$.
- So $D \in C((X \uplus [A]) \sqcup [B])$. Since $A \vdash B$, $B \in C([A])$.
- By Fact, $D \in C((X \uplus [A]) \sqcup [A])$.
- But $(X \uplus [A]) \sqcup [A] = X \uplus A$. So $D \in C(X \uplus [A])$; contradiction.

Any closure on multisets must grapple with:

Liars and closures:

If $\neg T\langle\lambda\rangle \in C([\lambda])$, and
$\bot \in C([\lambda, \neg T\langle\lambda\rangle])$, then
$\bot \in C([\lambda])$.

$$\lambda \vdash \neg T\langle\lambda\rangle$$
$$\lambda, \neg T\langle\lambda\rangle \vdash \bot$$
$$\lambda \vdash \bot$$

From transitivity to contraction

Summary

What's the lesson here?

Motivations for transitivity also push for contraction.

This is because they require
conditions that look like cut proper,
but are distinct:
cautious cut, or the closure condition.

Mere cut shouldn't satisfy anyone who wants transitivity.

So whether contraction or cut is the culprit,
the best arguments for transitivity must be mistaken.

But then why accept cut?