Vagueness and Probabilistic Consequence

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Polysemy, Ad Hoc Concepts, and Vagueness
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PLEXUS
PHILOSOPHICAL, LOGICAL, AND EXPERIMENTAL ROUTES TO SUBSTRUCTURALITY

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Section 1

Introduction

Vagueness and Polysemy

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Even if you fix the comparison class, however, there remains room for different interpretations of "tall".

Vagueness and Ambiguity

Vagueness is a form of structural ambiguity, but we should distinguish:

- Lexical ambiguity: same word expresses two or more concepts that are **distant/disconnected** from each other in conceptual space ("bat") [mammal bat, baseball bat]
- Lexical vagueness: same word expresses open meaning that can be precisified into a multiplicity of **closely related/connected concepts** in conceptual space ("tall") [taller than 180, taller than 180.1,....]

Pinkal 1995, Egré 2018

Vagueness and Paradox

One of the hallmarks of vague predicates like "tall" is their susceptibility to paradox, exposed in the ancient arguments of the Bald Man and the Sorites, both credited to Eubulides of Miletus.

(2) If n grains of wheat make a heap, then $n - 1$ grains make a heap.

(3) If a person with n hairs on their head is bald, then a person with $n + 1$ hairs is bald.

(4) If is a person is tall, then a person just 1mm shorter is tall too.

The application of vague predicates exhibit what C. Wright (1976) called *tolerance* to small changes.

Different solutions to the sorites

- Epistemicism: reject the validity of tolerance, and admit that there is a precise cutoff point separating heaps and non-heaps, or bald and non-bald. (Williamson 1992, Sorensen 2001)
- Strict-Tolerantism: accept the validity of tolerance, and revise the logic to make it tolerance-sensitive. (Cobreros et al. 2012)

Strict-Tolerant logic

Basic idea: vague predicates can be interpreted in a broad/tolerant sense, or in a narrow/strict sense

Dana is tall.

(5) a. Dana is [strictly] tall. [Dana's height is above 190cm]
    b. Dana is [tolerantly] tall. [Dana's height is above 170cm]

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If n grains make a heap, then $n-1$ make a heap.

(6) Either n grains do not make a heap [strictly] or $n-1$ grains make a heap [tolerantly]

Probabilistic Vagueness: A third way?

In this paper, we are interested in the exploration of a third family of approaches, which propose to handle tolerance conditionals probabilistically, as principles of inductive/risky reasoning.

This approach, we reckon, was pioneered by Borel 1907 in a short essay on the sorites paradox, in which Borel argued that these tolerance principles should be assigned degrees of probability, rather than simple yes or no values.

It is also represented in work by Edgington (1997), and by Lassiter and Goodman (2017)

Borel on Vagueness

"The questions [surrounding the sorites] are badly put, if one requires a yes or no answer; the true answer is a coefficient of probability" (Borel 1907)

Symmetric consequence relations

If we except Edgington who connected her approach to Adams's logic of conditionals, these probabilistic treatments have not resulted in a well-defined probabilistic logic of vague predicates.

Our goal here: look at a family of probabilistic consequence relations called symmetric consequence relations in a recent joint paper (Egré and Ripley 2025). These consequence relations differ from the idea of merely preserving probability over a threshold between premises and conclusions.

Instead, given a threshold $\alpha \leq 1$, symmetric consequence says that a sentence B is a consequence of some premises $A_1, ..., A_n$ provided every probability distribution that assigns the A_i a probability above α necessarily assigns B a probability above a symmetric threshold $1 - \alpha$.

Modeling Tolerance

So far, however, these symmetric consequence relations were investigated only for a classical propositional language.

Here, we propose to study them over a predicate language equipped with similarity relations, as in Cobreros et al. 2012, in order to adequately model tolerance principles.

We will show that these symmetric logics give us a novel treatment of the sorites paradox.

Like ST, these logics can validate the tolerance principles without triviality. However, in comparison to ST we will also see that they give us a substantially different account of the ambivalence felt in borderline cases of vague predicates, one that arguably preserves more classicality than ST.

Section 2

The sorites

The tolerance principle

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The tolerance principle has all the hallmarks of a principle of inductive reasoning:

- Our confidence in the principle is typically very high, or sufficiently high to allow us to rely on it for various inferences.
- Our confidence in the principle is not perfect, otherwise sorites arguments would lead to paradox.
- Our confidence in the principle can be increased or decreased depending on how similar consecutive items in a sorites sequence are taken to be, but also depending on their position in the sorites (see Egré, Ripley, Verheyen 2019, "The sorites in psychology").

A probabilistic take on tolerance

Various approaches exist that offer a probabilistic view of tolerance (including Borel (1907), Edgington (1997), Lassiter (2011), Egré (2011), Lassiter and Goodman (2017))

They are applicable to the material conditional version of the sorites, which we also use as a baseline (usual argument: negation + conjunction suffice to build a sorites)

The material reading

Basic idea: to be tall (within a comparison class) is to be taller than a threshold (viz. Kennedy 2007), but the threshold can be variable or uncertain.

Assume a probability distribution Pr on thresholds, then:

$$Pr(Fa_{n+1} \supset Fa_n) = Pr(\neg(Fa_{n+1} \wedge \neg Fa_n)) = 1 - Pr(Fa_{n+1} \wedge \neg Fa_n)$$

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$$Pr(Fa_{n+1} \wedge \neg Fa_n) = \int_{h(a_n)}^{h(a_{n+1})} Pr(\theta)d\theta$$

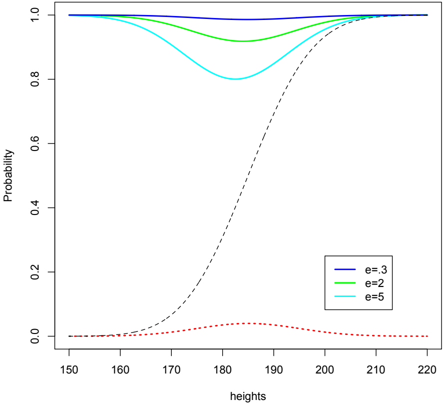
This is the probability of the threshold falling between a_{n+1} and a_n

Step between consecutive items

Let us understand: $y = a_n$ and $x = a_{n+1}$ to mean

$$h(x) = h(y) + e$$

We can calculate $Pr(Fx \supset Fy)$ for different values of e (the tolerance step)



Red: $Pr(\theta)$; Black: $Pr(Fx)$; Other: $Pr(Fx \supset Fy)$, with $h(x) - h(y) = e$

Observations

The approach vindicates the inductive take on the sorites paradox:

- the probability of (each instance of) material tolerance can come out very high, but cannot be uniformly 1 on each instance
- The probability of material tolerance gets higher or lower as the step between consecutive items decreases or increases
- the probability of material tolerance dips for instances closer to the center of the threshold distribution.

Our problem

We see that even when step sizes along a sorites sequence are relatively large, the probability of each tolerance conditional remains quite large, above 0.8 in our example.

Our problem: how can we incorporate this probabilistic account of tolerance as high-probability principles within an account of logical consequence that takes probability into account?

Before answering this, let us look at the strict-tolerant account of vague predicates, which is qualitative rather than probabilistic.

The sorites in ST

ST logic validates the tolerance principle in various forms, including:

(8) $Fa, a \sim_F b \models Fb$

(9) $a \sim_F b \models Fa \supset Fb$

(10) $\models \forall x \forall y (Fx \wedge x \sim_F y \supset Fy)$

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However, ST rests on qualitative, coarse-grained models and cannot tell us much about the relation between similarity and probability

From ST to Symmetric consequence

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They differ from ST and from Fuzzy Logics in being based on classical probability distributions, rather than standard many-valued truth functions.

First we will present symmetric consequence over a simplified, propositional language. Then we will constrain the models to deal with the sorites.

Section 3

Symmetric consequence

Language, Arguments, Models

- Language: countable infinite set of atoms, $\neg, \vee, \wedge, \top, \bot$ as main connectives
- Argument: $\Gamma \succ \Delta$, a pair of finite sets Γ, Δ of sentences of the language.
- Model: a classical probability assignment.

High sets and Low sets

A high set is a set α such that $\{1\} \subseteq \alpha \subseteq (0,1]$ and whenever $x \in \alpha$ and $x \leq y$, then $y \in \alpha$.

High sets stand for "good enough" probabilities

Every high set α has a corresponding low set $\overline{\alpha}$, defined as the set of values $1 - x$ for $x \in \alpha$.

Symmetric consequence

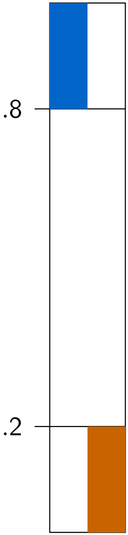
For a given high set α, recall that $\overline{\alpha} = \{x \in [0,1] \mid 1 - x \in \alpha\}$, and then:

α-symmetric consequence

A probability distribution Pr is an α-symmetric counterexample to $\Gamma \succ \Delta$ iff:
$Pr[\Gamma] \subseteq \alpha$ and $Pr[\Delta] \subseteq \overline{\alpha}$.

And $\Gamma \succ \Delta$ is α-symmetric valid (written $\vDash_\alpha \; \Gamma \succ \Delta$ or $\Gamma \vDash_\alpha \; \Delta$) iff:
no probability distribution is an α-symmetric counterexample to it.

Adjunction



We have $p, q \models_{[.8,1]} p \wedge q$; when $Pr(p), Pr(q) \geq .8$, the lowest $Pr(p \wedge q)$ can be is .6.

Still, $p, q, r, s \not\models_{[.8,1]} p \wedge q \wedge r \wedge s$.

What are these logics like?

Adjunctions:

Let P_n be the first n atomic sentences, and let $Conj_n$ be $P_n \succ \bigwedge P_n$.

Then $\vDash_\alpha$ validates $Conj_n$ iff $\alpha \subseteq (\frac{n}{n+1}, 1]$.

Modus ponenseses (Sorites in the medieval sense)

Let MP_n be $p_1, p_1 \supset p_2, \ldots, p_{n-1} \supset p_n \succ p_n$.

Then $\vDash_\alpha$ validates MP_n iff $\alpha \subseteq (\frac{n}{n+1}, 1]$.

Conditional Introduction

If $\Gamma, A \vDash_\alpha B, \Delta$ then $\Gamma \vDash_\alpha A \supset B, \Delta$, no matter what α is

Reflexivity and Transitivity

Reflexivity

α-symmetric consequence is reflexive iff $.5 \notin \alpha$.

Transitivity

α-symmetric consequence is transitive iff $.5 \in \alpha$ or $\alpha = \{1\}$.

So: among α-symmetric consequence relations, only one is fully Tarskian: the limit case of $\{1\}$-symmetric consequence, which is classical consequence.

For our applications: we are mainly thinking of the reflexive, nontransitive symmetric relations.

Section 4

Handling Similarity

Probabilistic closeness

Given a domain of objects and a predicate F, we take as a given which objects are F-similar (for instance because we set a threshold on JNDs, see Luce 1959), and we let Sim_F be the set of pairs of F-similar objects.

Basic idea: model a notion of probabilistic closeness (Smith 2008 on the fuzzy case; Borel 1907, Egré 2011 on probabilities): similar items in the F-relevant features should give rise to closely related judgments on average, so to closely related probabilities of judging F.

Language and Models

Language: we now have a predicate language, with a vague predicate F and a similarity predicate $\sim_F$

Models: A probabilistic model now assign probabilities to closed formulae of the language, and interprets $\sim_F$ by the relation Sim_F.

Similarity and Probability

Given a probability distribution Pr, a set Sim_F of pairs of individuals (understood as the extension of the F-similarity relation), and a real $\varepsilon \in [0, 1]$.

Similarity-respecting Probability

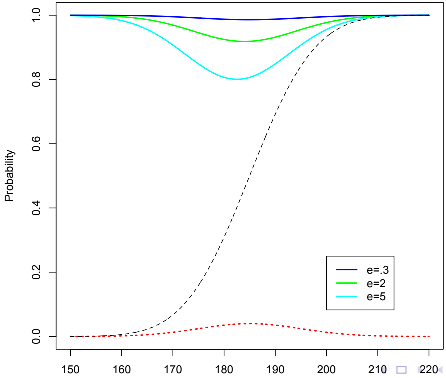
We say that Pr respects Sim_F to within ε iff
for all a, b if $\langle a, b \rangle \in Sim_F$, then $|Pr(Fa) - Pr(Fb)| \leq \varepsilon$.

Fact

If Pr respects Sim_F to within ε, and $Sim'_F \subseteq Sim_F$ and $\varepsilon \leq \varepsilon'$, then Pr respects Sim'_F to within ε'.

Height difference and Probabilistic Difference

We assume *$Sim_T all$* to be based on how close two heights are (so within e). Our parameter ε differs from e, it concerns not a proximity in heights, but a proximity between probabilities of sentences relative to “tall”.



Similarity

Similarity in the language

We add $a \sim_F b$ to the language and stipulate that for every probability distribution Pr, $Pr(a \sim_F b) = 1$ iff $(a, b) \in Sim_F$.

Symmetric consequence over similarity-ε-respecting probability

We write $\Gamma \vDash^{Sim_F}_{\alpha,\varepsilon} \Delta$ iff for every probability distribution Pr that respects Sim_F to within ε, $\Gamma \vDash_{\alpha} \Delta$

Tolerance

1-step Tolerance

$Fa, a \sim_F b \vDash^{Sim_F}_{\alpha,\varepsilon} Fb$ iff

$\neg Fb, a \sim_F b \vDash^{Sim_F}_{\alpha,\varepsilon} \neg Fa$ iff

$\forall x \in \alpha, x - \varepsilon \notin \overline{\alpha}$.

n-step Tolerance

$Fa_0, a_0 \sim_F a_1, ..., a_{n-1} \sim_F a_n \vDash^{Sim_F}_{\alpha,\varepsilon} Fa_n$ iff

$\neg Fa_n, a_0 \sim_F a_1, ..., a_{n-1} \sim_F a_n \vDash^{Sim_F}_{\alpha,\varepsilon} Fa_0$ iff

$\forall x \in \alpha, x - n\varepsilon \notin \overline{\alpha}$.

Comparison with ST

Suppose an α, ε where $\forall x \in \alpha, x - \varepsilon \notin \overline{\alpha}$. Symmetric consequence validates the tolerance principle in argument form and sentential form, but cannot guarantee it as a universally quantified premise.

(11) $Fa, a \sim_F b \models^{Sim_F}_{\alpha,\varepsilon} Fb$

(12) $a \sim_F b \models^{Sim_F}_{\alpha,\varepsilon} Fa \supset Fb$

(13) $\not\models^{Sim_F}_{\alpha,\varepsilon} \forall x \forall y (Fx \wedge x \sim_F y \supset Fy)$

Borderline contradictions?

Let Γ say that there is a 10-step sorites;
that is, let $\Gamma = \{Fa_0, \neg Fa_{10}\} \cup \{a_i \sim_F a_{i+1} \mid 0 \leq i < 10\}$.
By adjusting α, ε, we get different verdicts about Γ.

If we don't have tolerance at all, nothing interesting seems to happen.
If we have tolerance, but not yet 5-step tolerance,
we get the usual kind of nontransitivity.
If we have at least 10-step tolerance,
no probability assignment can bring all of Γ into α,
and so $\Gamma \models^{Sim_F}_{\alpha,\varepsilon} B$ for any B.

In the middle, where we have at least 5-step tolerance but not yet 10-step,
Γ is not explosive, and an interesting phenomenon emerges.

Borderline contradictions?

Since a_5 is 5 steps from a_0,
if α, ε give us 5-step tolerance then we have $\Gamma \models^{Sim_F}_{\alpha,\varepsilon} Fa_5$.

But a_5 is also 5 steps from a_{10},
so in this case we also have $\Gamma \models^{Sim_F}_{\alpha,\varepsilon} \neg Fa_5$.

And since Γ is not explosive (and the probabilities are all classical)
we have $\Gamma \not\models^{Sim_F}_{\alpha,\varepsilon} Fa_5 \wedge \neg Fa_5$.

The setup entails two things that are contradictory,
but it does not entail any contradiction.

Conclusions and Further Work

- We handle the sorites probabilistically using mixed consequence in a way that bears a similarity with ST, but yields a distinct family of logics

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- We handle the sorites probabilistically using mixed consequence in a way that bears a similarity with ST, but yields a distinct family of logics
- Ambivalence about borderline cases is captured, but unlike in ST, borderline contradictions are not accepted qua conjunctions (more palatable to the classically-minded / less appealing to the nonclassically-minded)
- Investigations into nonclassical probability needed if we want to assign borderline contradictions nonzero probability.

THANK YOU

Map of symmetric consequence relations

Extreme cases:

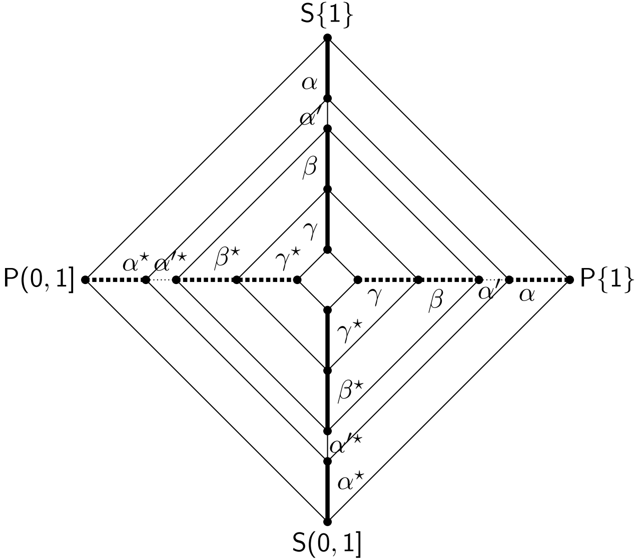
$\vDash_{\{1\}}$ is exactly classical logic,

$\Gamma \vDash_{(0,1]} \Delta$ iff there is a contradiction in Γ or a tautology in Δ.

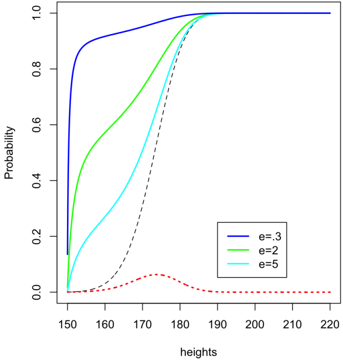
Ordering:

When $\alpha \subseteq \beta$, it's immediate that $\vDash_\beta \subseteq \vDash_\alpha$;
every α-symmetric counterexample just is a β-symmetric one as well.

Preservation and Symmetric consequence



Conditional probability of Ta_n given Ta_{n+1}



Red: $Pr(\theta)$; Black: $Pr(Tx)$; Other: $Pr(Ty|Tx, h(x) = h(y) + e)$

$Ta_{n+1} \supset Ta_n$ with uniform prior