Setting the bounds

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Historical prelude

Gentzen's sequents

Gentzen's dissertation was a landmark for proof theory

Key notions introduced: natural deduction and sequent calculus

Sequent calculus for classical logic worked on things of the form $\Gamma \Rightarrow \Delta$, where Γ and Δ are finite lists of formulas.

Gentzen:

"The sequent $A_1, \ldots, A_n \Rightarrow B_1, \ldots, B_m$ has the same meaning as the formula $(A_1 \wedge \ldots \wedge A_n) \supset (B_1 \vee \ldots \vee B_n)$.'

By organizing his calculus in this way,
Gentzen was able to do lots of nice things.

Gentzen seemed to think this was all a technical trick.

But what if it's more than that?

From bounds to meaning

Multiple conclusions

A sequent $\Gamma \Rightarrow \Delta$ can be seen as representing an argument with premises Γ and conclusions Δ.

This can take a bit of practice;
recall that the conclusions are disjunctively combined.

When is such an argument valid?

The key idea here is from Restall:

Restall (2005, 2008, 2009, 2013):

An argument is valid iff:
asserting all its premises and denying all its conclusions clashes.

Other phrasing: 'out of bounds', 'incoherent', 'self-defeating'.

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Example:

Asserting and denying the same thing is out of bounds.

So $A \vdash A$.

Example:

Asserting ‘Auckland is bigger than Wellington’ and ‘Wellington is bigger than Palmy’ while denying ‘Auckland is bigger than Palmy’ is out of bounds.

So $A >> W, W >> P \vdash A >> P$.

A position is a collection of assertions and denials.

It is positions that are in or out of bounds.

$\Gamma \Rightarrow \Delta$ is valid iff the position that asserts the Γs and denies the Δs is out of bounds.

This gives a way to understand Gentzen's (and others') sequent rules:

Some example rules

KL: $$\frac{\Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta}$$

KR: $$\frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A}$$

∧L: $$\frac{A/B, \Gamma \Rightarrow \Delta}{A \wedge B, \Gamma \Rightarrow \Delta}$$

∧R: $$\frac{\Gamma \Rightarrow \Delta, A \qquad \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \wedge B}$$

TL: $$\frac{A, \Gamma \Rightarrow \Delta}{T\langle A \rangle, \Gamma \Rightarrow \Delta}$$

TR: $$\frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, T\langle A \rangle}$$

Rules 3–6 give the meanings of $\wedge$, T.

All that's well and good (let's suppose).

But what's a clash in the first place?

From bounds to meaning

Vocabulary-independent

Recall Gentzen:

"The sequent $A_1, \ldots, A_n \Rightarrow B_1, \ldots, B_m$ has the same meaning as the formula $(A_1 \wedge \ldots \wedge A_n) \supset (B_1 \vee \ldots \vee B_n)$.'

We don't want this kind of approach to clashes.

Adding vocabulary to the sequent
—$\wedge$, $\vee$, $\supset$, whatever—
is a bad idea, for three reasons.

First:

There are perfectly sensible applications of this approach to languages that lack $\wedge$, $\vee$, $\supset$, etc—eg English.

Possible response: see such languages as fragments of fuller languages that do contain the needed vocabulary?

But that's not always possible.

Second:

Infinite collections of premises or conclusions don't require infinitary connectives.

Again, seeing these as a fragment of fuller languages
is not always possible.

Third:

The sequent rules, interpreted via clashes, explain the meanings of $\wedge$, $\vee$, $\supset$, etc.

Dragging their meanings into the interpretation would give an explanatory circle.

From bounds to meaning

Possible truth?

Another nonstarter:

A sequent $\Gamma \Rightarrow \Delta$ clashes when it's impossible for all the Γs to be true while all the Δs are false.

Trouble:

Whether it's possible for all the Γs to be true while the Δs are false depends on what the Γs and Δs mean.

So this would again result in an explanatory circle.

From bounds to meaning

What sets the bounds?

Where do the bounds come from, then?

Not from implication,
not from possible truth-and-falsity,
so . . .

The bounds are a social kind:
they are created and sustained
by the place they occupy in our social practices.

Some norms on assertion and denial:

— Assert only what's true
— Deny only what's false

— Assert only what you have warrant for
— Deny only what you have warrant against

— Assert or deny only what's relevant

⋮

Norms involving the bounds:

— Don’t adopt a position that’s out of bounds

— What’s out of bounds is discountable

Discountable: it’s ok to ‘leave one’s flank open’ to risks from this angle.

Three characteristic responses to clashes:

- Reinterpretation
- Clarification
- Dismissal

Example clash:

Someone asserts both:
‘Napoleon died in 1821’
‘Napoleon organized a coup in 1851’

In a context where we’re not taking zombie Napoleon seriously, this is probably a clash.

Three responses:

Reinterpret:	‘They must be talking about two different Napoleons’
Clarify:	‘But I thought you said he had died in 1821?’
Dismiss:	‘This asshole is just talking nonsense’

These responses are not exclusive, and they shade into each other.

They all mark a standing back from what’s been claimed.

Discountability:

Given that Napoleon died in 1821,
there's no risk that he led a coup in 1851.

Nor is there any risk he'll lead a coup tomorrow.

Discountability works in ‘what-ifs’ too:

What if Palmy were bigger than Auckland?
Then there’d be no risk of it being smaller than Wellington.

Words have the meanings we give them;
we give meaning by treating things as clashing or not.

(Reinterpreting, clarifying, dismissing, discounting)

This is a stance approach to the bounds:
the crucial notion is treating something as a clash.

A natural way to think about social kinds.

Rumfitt's objection

Moore's paradox

Rumfitt (2008) offers two related objections.

One is patterned after Moore's paradox:

Rumfitt:

"A thinker who accepts that it is raining but denies that he accepts that it is raining will be making a mistake as to the facts. But the statement 'He accepts that it is raining' is not a consequence of the statement 'It is raining'.... This sort of case is a problem for those who seek to explicate consequence in normative terms."

Suppose someone asserts 'It's raining'
and denies 'I've asserted that it's raining'.

They've surely done something wrong.

Rumfitt's objection

Lack of grounds

The second is similar:

Rumfitt:

"The mental state that consists of accepting that there will never be grounds for accepting or rejecting 'There is a god', while rejecting that very statement, is self-defeating. But 'There is a god' is in no sense a consequence of 'There will never be sufficient grounds for accepting or rejecting "There is a god"'."

Suppose someone asserts 'There will never be sufficient grounds for asserting or denying that there is a god' and denies 'There is a god'.

Again, they've surely done something wrong.

Rumfitt's objection

Answering Rumfitt

Both cases violate norms.
But neither violates the bounds.

Moore paradox: violates 'assert only what's true'

Groundlessness: violates 'assert only what's warranted'

Discountability shows the difference:

Moore:

Can't discount that it's raining but that I didn't assert it.
(This happens all the time!)

Grounds:

Can't discount that there's no god and
no grounds for this.
(This may well be the case!)

Clashes are a particular social kind.

There is a norm: ‘don’t adopt a clashing position’

This doesn’t mean anything that violates a norm is a clash.

- Multiple-conclusion consequence can be understood in terms of clashes.
- If clashes are understood the right way, this gives foundations for a theory of both consequence and meaning.
- Clashes are a social kind.
- Two key norms: don't clash, and discount clashes.