"Classical logic is not so much wrong—if by this word we mean that it ascribes disputable properties to the logical constants it deals with—as ambiguous: its connectives are ill-defined inasmuch as they have multiple meanings" (Paoli 2007)

"There is no need to give up any compelling inferential principle of classical logic—only to recognize that bad things can happen when principles holding of different connectives are used, in the course of a derivation, as holding of the same ambiguous connective…" (Mares & Paoli 2014)

Recapture via ambiguity

$$\begin{array}{ll} \text{Atoms } At & := p, q, r, \text{etc.} \\ \text{Classical } (\mathcal{L}_{\mathrm{C}})\ C & := At \mid \neg C \mid \mathrm{T} \quad \mid C \wedge C \\ \text{Linear } (\mathcal{L}_{\mathrm{L}})\ L := & := At \mid \neg L \mid \top \mid \mathsf{t} \mid L \sqcap L \mid L \otimes L \end{array}$$

Other connectives can be defined in usual ways : in $\mathcal{L}_{\mathrm{C}}$: $\vee$; $\rightarrow$; F; and in $\mathcal{L}_{\mathrm{L}}$: $\sqcup, \parr$; $\sqsupset, \multimap$; $\bot, \mathsf{f}$.
All sequents are finite multiset-finite multiset.

Classical and linear languages

Id: $\dfrac{}{[p \succ p]}$ D: $\boxed{\dfrac{[\Gamma \succ \Delta]}{[\Gamma', \Gamma \succ \Delta, \Delta']}}$ ¬L: $\dfrac{[\Gamma \succ \Delta, A]}{[\neg A, \Gamma \succ \Delta]}$ ¬R: $\dfrac{[A, \Gamma \succ \Delta]}{[\Gamma \succ \Delta, \neg A]}$

⊓L: $\dfrac{[A/B, \Gamma \succ \Delta]}{[A \sqcap B, \Gamma \succ \Delta]}$ ⊓R: $\dfrac{[\Gamma \succ \Delta, A] \qquad [\Gamma \succ \Delta, B]}{[\Gamma \succ \Delta, A \sqcap B]}$ ⊗L: $\dfrac{[A, B, \Gamma \succ \Delta]}{[A \otimes B, \Gamma \succ \Delta]}$ ⊗R: $\dfrac{[\Gamma \succ \Delta, A] \qquad [\Gamma' \succ \Delta', B]}{[\Gamma, \Gamma' \succ \Delta, \Delta', A \otimes B]}$

⊤R: $\dfrac{}{[\Gamma \succ \Delta, \top]}$ tL: $\dfrac{[\Gamma \succ \Delta]}{[\mathsf{t}, \Gamma \succ \Delta]}$ tR: $\dfrac{}{[\succ \mathsf{t}]}$

Two systems: MAAL is the full system; MALL is MAAL minus D. Cut's admissible in both.

Affine logic MAAL and linear logic MALL

C	$\gamma^-(C)$	$\gamma^+(C)$	$o^-(C)$	$o^+(C)$
p	p	p	$\mathsf{t} \sqcap p$	$\mathsf{f} \sqcup p$
T	t	$\top$	t	$\top$
$\neg A$	$\neg\gamma^+(A)$	$\neg\gamma^-(A)$	$\neg o^+(A)$	$\neg o^-(A)$
$A \wedge B$	$\gamma^-(A) \otimes \gamma^-(B)$	$\gamma^+(A) \sqcap \gamma^+(B)$	$o^-(A) \otimes o^-(B)$	$o^+(A) \sqcap o^+(B)$

Fact (Grišin): $\Gamma \vdash_{\mathrm{CL}} \Delta$ iff $\gamma^-[\Gamma] \vdash_{\mathrm{MAAL}} \gamma^+[\Delta]$.
Fact (Ono): $\Gamma \vdash_{\mathrm{CL}} \Delta$ iff $o^-[\Gamma] \vdash_{\mathrm{MALL}} o^+[\Delta]$.

Translations

"[I]f we confine ourselves to the classical tautologies that play a role in the known versions of the paradoxes, you do not need to replace propositional variables in order to get to a theorem of [MALL]" (Mares & Paoli 2014)

ϕ	$\gamma^+(\phi)$	MALL theorem?
$p \vee \neg p$	$p \parr \neg p$	✓
$\neg(p \wedge \neg p)$	$\neg(p \otimes \neg p)$	✓
$(p \rightarrow (p \rightarrow q)) \rightarrow (p \rightarrow q)$	$(p \sqsupset (p \sqsupset q)) \multimap (p \multimap q)$	×
$(p \wedge (p \rightarrow q)) \rightarrow q$	$(p \otimes (p \sqsupset q)) \multimap q$	×

Using γ with MALL?

L	p	$\top$	t	$\neg B$	$B \sqcap C$	$B \otimes C$
$\beta(L)$	p	T	T	$\neg\beta(B)$	$\beta(B) \wedge \beta(C)$	$\beta(B) \wedge \beta(C)$

Blurring translation from $\mathcal{L}_{\mathrm{L}}$ to $\mathcal{L}_{\mathrm{C}}$

Id: $\dfrac{}{[p \succ p]}$ | D: $\boxed{\dfrac{[\Gamma \succ \Delta]}{[\Gamma', \Gamma \succ \Delta, \Delta']}}$ | ¬L: $\dfrac{[\Gamma \succ \Delta, A]}{[\neg A, \Gamma \succ \Delta]}$ | ¬R: $\dfrac{[A, \Gamma \succ \Delta]}{[\Gamma \succ \Delta, \neg A]}$

$\wedge L^{\sqcap}$: $\boxed{\dfrac{[A/B, \Gamma \succ \Delta]}{[A \wedge B, \Gamma \succ \Delta]}}$ | $\wedge R^{\sqcap}$: $\dfrac{[\Gamma \succ \Delta, A] \quad [\Gamma \succ \Delta, B]}{[\Gamma \succ \Delta, A \wedge B]}$ | $\wedge L^{\otimes}$: $\dfrac{[A, B, \Gamma \succ \Delta]}{[A \wedge B, \Gamma \succ \Delta]}$ | $\wedge R^{\otimes}$: $\boxed{\dfrac{[\Gamma \succ \Delta, A] \quad [\Gamma' \succ \Delta', B]}{[\Gamma, \Gamma' \succ \Delta, \Delta', A \wedge B]}}$

$TR^{\top}$: $\dfrac{}{[\Gamma \succ \Delta, \top]}$ | TL^{t}: $\dfrac{[\Gamma \succ \Delta]}{[\top, \Gamma \succ \Delta]}$ | TR^{t}: $\boxed{\dfrac{}{[\succ \top]}}$

Two classical systems: FULL is the full system; G3ISH is the same without the single-boxed rules.
These are both sound and complete for classical logic, and have the same derivable rules.

Two nonclassical systems: NOD is FULL minus D; and NOD$^-$ is NOD minus the single-boxed rules.
These differ: $p \wedge q \vdash_{\text{NOD}} q$, but $p \wedge q \nvdash_{\text{NOD}^-} q$. Neither is classical: $p, q \nvdash_{\text{NOD}} q$.

Systems for the classical language

Recall: (Grišin) $\Gamma \vdash_{\text{CL}} \Delta$ iff $\gamma^-(\Gamma) \vdash_{\text{MAAL}} \gamma^+(\Delta)$
Proof: apply $\gamma^{\pm}$ to a G3ISH derivation of $[\Gamma \succ \Delta]$, get a MAAL derivation of $[\gamma^-[\Gamma] \succ \gamma^+[\Delta]]$;
apply β to a MAAL derivation of $[\gamma^-[\Gamma] \succ \gamma^+[\Delta]]$, get a G3ISH derivation of $[\Gamma \succ \Delta]$.

New fact: $\Gamma \vdash_{\text{NOD}^-} \Delta$ iff $\gamma^-(\Gamma) \vdash_{\text{MALL}} \gamma^+(\Delta)$ *Proof*: same.

The Grišin translations

Claim: Where $\vdash$ captures validity in $\mathcal{L}_1$ and $\beta : \mathcal{L}_1 \to \mathcal{L}_2$ captures the conflation of $\mathcal{L}_1$ in $\mathcal{L}_2$,
then $\vdash^{\beta}$ captures validity in $\mathcal{L}_2$, where $\Gamma \vdash^{\beta} \Delta$ iff there are Γ', Δ' such that $\Gamma' \vdash \Delta'$ and $\beta[\Gamma'] = \Gamma$ and $\beta[\Delta'] = \Delta$.

(From Ripley 2017, "Vagueness is a kind of conflation" and Ripley 2018, "Blurring")

A view of conflated validity

New fact: $\Gamma \vdash_{\text{CL}} \Delta$ iff there are Γ', Δ' such that:
$\Gamma' \vdash_{\text{MAAL}} \Delta'$ and $\beta[\Gamma'] = \Gamma$ and $\beta[\Delta'] = \Delta$.

New fact: $\Gamma \vdash_{\text{NOD}} \Delta$ iff there are Γ', Δ' such that:
$\Gamma' \vdash_{\text{MALL}} \Delta'$ and $\beta[\Gamma'] = \Gamma$ and $\beta[\Delta'] = \Delta$.

Proof of these facts: apply β to an MAAL/MALL derivation of $[\Gamma' \succ \Delta']$, get a FULL/NOD derivation of $[\beta[\Gamma'] \succ \beta[\Delta']]$;
and for any FULL/NOD derivation of $[\Gamma \succ \Delta]$, *there is some way* to pull it apart into an appropriate MAAL/MALL derivation.
(Each connective occurrence in $[\Gamma \succ \Delta]$ is introduced once in the derivation; choose its sharpening depending on how.)

Recapture via blurring

Lemma: If $[\Gamma \succ \Delta]$ is derivable in FULL, then it has a derivation in FULL with all dilutions at the end.

Proof: Permute dilutions downward.
If the diluted formulas are all side formulas for the next rule, then just dilute them in one step later.
If at least one is principal, the next rule is a ¬ or ∧ rule.

If it is anything but $\wedge L^{\otimes}$, or if it is $\wedge L^{\otimes}$ but both conjuncts are diluted in,
then dilution alone is enough to reach the same result.

Finally, if it is $\wedge L^{\otimes}$ and only one conjunct is diluted in, then do this:

$$\wedge L^{\otimes}: \dfrac{\text{D: } \dfrac{[\Gamma, A/B \succ \Delta]}{[\Gamma, \Gamma', A, B \succ \Delta, \Delta']}}{[\Gamma, \Gamma', A \wedge B \succ \Delta, \Delta']} \quad \Rightarrow \quad \text{D: } \dfrac{\wedge L^{\sqcap}: \dfrac{[\Gamma, A/B \succ \Delta]}{[\Gamma, A \wedge B \succ \Delta]}}{[\Gamma, \Gamma', A \wedge B \succ \Delta, \Delta']}$$

Permuting dilution

New fact: if $[\Gamma \succ \Delta]$ is minimally classically valid, then $\Gamma \vdash_{\text{NOD}} \Delta$

Proof: By completeness, there is a FULL proof of $[\Gamma \succ \Delta]$; take it and permute all dilutions to the end.
Since there is no FULL proof of any subsequent, this must be a NOD proof of $[\Gamma \succ \Delta]$.

Recapture for minimal validities