Position-theoretic semantics and entailment

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Position-theoretic semantics

The first central notion of this talk is the position.

A position $[\Gamma \succ \Delta]$ is any pair of sets of sentences,
with Γ the sentences asserted and Δ the sentences denied.

(Sentences, assertion, denial, all taken for granted here.)

We track each other's evolving positions in conversation, and our conversational moves depend on this.

('No, I didn't eat it' is only an appropriate response to someone who asserted that you ate it.)

The second central notion is disagreement between positions.

Positions P and Q disagree when
one person's adopting P and another's adopting Q
would constitute a disagreement between those people.

I'll write $P \frown Q$
to indicate P disagrees with Q.

Some boring notation:

Subposition:

$[\Gamma \succ \Delta] \sqsubseteq [\Sigma \succ \Theta] \quad \text{iff}_{df} \quad \Gamma \subseteq \Sigma \text{ and } \Delta \subseteq \Theta$

Position union:

$[\Gamma \succ \Delta] \sqcup [\Sigma \succ \Theta] \quad =_{df} \quad [\Gamma \cup \Sigma \succ \Delta \cup \Theta]$

Some more interesting notation:

Self-disagreement:

$[\Gamma \vdash \Delta] \quad \text{iff}_{df} \quad [\Gamma \succ \Delta] \frown [\Gamma \succ \Delta]$ $\qquad P^{\vdash} \quad \text{iff}_{df} \quad P \frown P$

Little positions:

$+\phi \quad =_{df} \quad [\phi \succ]$ $\qquad -\phi \quad =_{df} \quad [\succ \phi]$

Disagreement range / equivalence:

$\widehat{P} \quad =_{df} \quad \{R | P \frown R\}$ $\qquad P \simeq Q \quad \text{iff}_{df} \quad \widehat{P} = \widehat{Q}$

Assumption 1: monotonicity

If $P \sqsubseteq P'$ and $Q \sqsubseteq Q'$ and $P \frown Q$,
then $P' \frown Q'$.

It follows that if $P \frown Q$, then $(P \sqcup Q)^{\vdash}$.

Assumption 2: disaggregation

If $(P \sqcup Q)^{\vdash}$, then $P \frown Q$.

With both assumptions in place, we can reduce disagreement to self-disagreement:

$$P \frown Q \quad \text{iff} \quad (P \sqcup Q)^{\vdash}$$

This is of technical convenience, but shouldn't be overstated.

It is disagreement is directly tied to conversational actions, and we might want to question these assumptions.

We can build a compositional semantics in these terms,
rather than, eg, truth, falsity, warrant, inference, etc.

Work by way of assertion and denial conditions,
understood as conditions under which assertions and denials
disagree.

Example: negation

$$\frac{[\Gamma \vdash \Delta, A]}{[\neg A, \Gamma \vdash \Delta]} \qquad \frac{[A, \Gamma \vdash \Delta]}{[\Gamma \vdash \Delta, \neg A]}$$

Example: negation

$$\begin{array}{c} [\Gamma \vdash \Delta, A] \\ \hline\hline [\neg A, \Gamma \vdash \Delta] \end{array} \qquad \begin{array}{c} [A, \Gamma \vdash \Delta] \\ \hline\hline [\Gamma \vdash \Delta, \neg A] \end{array}$$

$$\begin{array}{c} [\Gamma \succ \Delta] \frown -A \\ \hline\hline [\Gamma \succ \Delta] \frown +\neg A \end{array} \qquad \begin{array}{c} [\Gamma \succ \Delta] \frown +A \\ \hline\hline [\Gamma \succ \Delta] \frown -\neg A \end{array}$$

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$$\widehat{+\neg A} = \widehat{-A} \qquad \widehat{-\neg A} = \widehat{+A}$$

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$$\widehat{+\neg A} = \widehat{-A} \qquad \widehat{-\neg A} = \widehat{+A}$$

$$-A \simeq +\neg A \qquad +A \simeq -\neg A$$

Example: conjunction

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \wedge B, \Gamma \vdash \Delta]} \qquad \frac{[\Gamma \vdash \Delta, A] \qquad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \wedge B]}$$

Example: conjunction

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$$\frac{[\Gamma \succ \Delta] \frown [A, B \succ]}{[\Gamma \succ \Delta] \frown +A \wedge B} \qquad \frac{[\Gamma \succ \Delta] \frown -A \qquad [\Gamma \succ \Delta] \frown -B}{[\Gamma \succ \Delta] \frown -A \wedge B}$$

Example: conjunction

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \wedge B, \Gamma \vdash \Delta]} \qquad \frac{[\Gamma \vdash \Delta, A] \qquad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \wedge B]}$$

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$$\widehat{+A \wedge B} = \widehat{[A, B \succ]} \qquad \widehat{-A \wedge B} = \widehat{-A} \cap \widehat{-B}$$

Example: conjunction

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \wedge B, \Gamma \vdash \Delta]} \qquad \frac{[\Gamma \vdash \Delta, A] \qquad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \wedge B]}$$

$$\frac{[\Gamma \succ \Delta] \frown [A, B \succ]}{[\Gamma \succ \Delta] \frown +A \wedge B} \qquad \frac{[\Gamma \succ \Delta] \frown -A \qquad [\Gamma \succ \Delta] \frown -B}{[\Gamma \succ \Delta] \frown -A \wedge B}$$

$$\widehat{+A \wedge B} = \widehat{[A, B \succ]} \qquad \widehat{-A \wedge B} = \widehat{-A} \cap \widehat{-B}$$

$$+A \wedge B \simeq [A, B \succ]$$

What's key in this approach is just
which positions disagree.

Models can show non-disagreement,
but the models aren't the point.

Proofs can show disagreement,
but the proofs aren't the point.

If disagreement reduces to self-disagreement,
then everything is settled by $\vdash$.

The formal tools of most direct use
are thus consequence-theoretic.

Consequence relations themselves matter,
not any particular way of determining them.

Can this do what we want a semantics to do?

Dowty, Wall, Peters 1981:

"In constructing the semantic component of a grammar, we are attempting to account...[for speakers'] judgements of synonymy, entailment, contradiction, and so on" (2, emphasis added).

Can this do what we want a semantics to do?

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A problem: entailment

There might seem to be an easy path to entailment:
say that Γ entails A iff $[\Gamma \vdash A]$

If some controversial assumptions hold,
I'll argue, this view is extensionally right.

But even if this is so, it's more or less a coincidence.

Steinberger (2011):

“Take the example of the classical theoremhood of the law of the excluded middle. [This] would have to be rendered as ‘It is incoherent to deny $A \vee \neg A$’. But surely this is not what is intended; even the intuitionist can happily agree that it is incoherent to deny (every instance of) $A \vee \neg A$. [We need] a way of expressing that $A \vee \neg A$ can always be correctly asserted” (353).

$[\Gamma \vdash A]$ just says:
$[\Gamma \succ A]$ disagrees with itself.

If anything, it's a prohibition, a ruling out.
But entailment should enable us to go on.

In the limiting case of empty Γ,
it should ensure that A is assertible.

$[\vdash A]$ doesn't do that.

Or: if truth and falsity are projections from assertion and denial,

then $[\Gamma \vdash A]$ just says:
Γ can't be true while A is false.

But entailment should connect truth to truth.

In the limiting case of empty Γ,
it should ensure that A is true.

$[\vdash A]$ doesn't do that.

A solution: implicit assertion

Recall that $P \simeq Q$ iff $\widehat{P} = \widehat{Q}$

As far as disagreements go,
equivalent positions are just the same.

This means an adopter of P has the same options open
for going on as an adopter of Q does.

Say that a position P implicitly asserts A iff:

$$P \simeq P \sqcup +A$$

A is implicitly asserted when adding a genuine assertion of A wouldn't add any new disagreements.

(Mutatis for implicit denial, but that won't play a role here.)

Implicit assertion is a broad notion, subsuming assertion proper.

$([A, \Gamma \succ \Delta] \sqcup +A$ is just $[A, \Gamma \succ \Delta]$ again.)

Implicit assertion can help us fill the role of entailment.

Write $P \Vdash^{+} A$ to indicate that P implicitly asserts A.

If $P \Vdash^{+} A$, when someone has already at least adopted P, they change nothing by going on to assert A.

Implicit assertion is monotonic:

If $P \sqsubseteq Q$ and $P \Vdash^{+} A$,
then $Q \Vdash^{+} A$ too.

Proof sketch:

Always $\widehat{Q} \subseteq \widehat{Q \sqcup +A}$, so we only need to show the converse.

Suppose, then, that $Q \sqcup +A \frown R$.

Since $P \sqsubseteq Q$, Q is $P \sqcup S$ for some S, so $P \sqcup S \sqcup +A \frown R$.

This means $P \sqcup +A \frown R \sqcup S$.

But P implicitly asserts A, so $P \frown R \sqcup S$.

Finally, this gives $P \sqcup S \frown R$, which is to say $Q \frown R$.

Implicit assertion has other structural properties we might expect of entailment.

For any A, we have $+A \Vdash^{+} A$

If $P \Vdash^{+} A$ and $+A \sqcup P \Vdash^{+} B$,
then $P \Vdash^{+} B$

Suppose our earlier theory of conjunction's assertion conditions:

$$\widehat{+A \wedge B} = \widehat{[A, B\succ]}.$$

Then $[A, B\succ] \Vdash^{+} A \wedge B$.

Here's why:

Suppose $[A, B, A \wedge B\succ] \frown R$; that is, that $+A \wedge B \sqcup [A, B\succ] \frown R$.

Then $+A \wedge B \frown R \sqcup [A, B\succ]$.

By the assertion conditions, $[A, B\succ] \frown R \sqcup [A, B\succ]$.

And so $[A, B\succ] \frown R$.

Any position asserting A and B implicitly asserts $A \wedge B$.

Or suppose that $[\succ] \Vdash^{+} A$.

If $[\succ]$ implicitly asserts A,
then every position does.

If this is so, assertions of A are free;
they close off no options at all, for anyone.

This is just what we were after:
a permissive notion.

Entailment and consequence

Getting to $[A, B \succ] \Vdash^{+} A \wedge B$ assumed only $\widehat{+A \wedge B} = \widehat{[A, B \succ]}$,
plus monotonicity and disaggregation.

This is not enough to show $[A, B \vdash A \wedge B]$;
we'd want in addition that $[A \wedge B \vdash A \wedge B]$,
which does not follow from anything so far.

(Entailment is reflexive, but consequence may not be!)

So we don't have an implication from $[\Gamma \succ \Delta] \Vdash^{+} A$ to $[\Gamma \vdash \Delta, A]$

There is also no implication in general from $[\Gamma \vdash \Delta, A]$ to $[\Gamma \succ \Delta] \Vdash^+ A$.

Suppose A is such that $+A \frown [\Gamma \succ \Delta]$ and $-A \frown [\Gamma \succ \Delta]$.
(Maybe $[\Gamma \succ \Delta]$ has it that A attributes a vague predicate to one of its borderline cases.)

And suppose there is some Q with $[\Gamma \succ \Delta] \not\frown Q$.

Then $[\Gamma \vdash \Delta, A]$ but $[\Gamma \succ \Delta] \nVdash^+ A$.

So in general $[\Gamma \vdash \Delta, A]$ and $[\Gamma \succ \Delta] \Vdash^{+} A$ are independent claims.

However, if disagreement obeys certain properties,
then these collapse.

If $[A \vdash A]$ and $[\Gamma \succ \Delta] \Vdash^{+} A$,
then $[\Gamma \vdash \Delta, A]$.

If you'd self-disagree by denying and asserting A,
then you'd self-disagree by denying A if you've implicitly asserted it.

If $[\Gamma \vdash \Delta, A]$ and $[A, \Gamma \vdash \Delta]$ implies $[\Gamma \vdash \Delta]$,

and $[\Gamma \vdash \Delta, A]$,
then $[\Gamma \succ \Delta] \Vdash^{+} A$.

If disagreeing with both $+A$ and $-A$ means self-disagreeing,
then if you disagree with $-A$ you've implicitly asserted it.

So if we assume $\vdash$ obeys identity and cut, then we get $[\Gamma \succ \Delta] \Vdash^{+} A$ iff $[\Gamma \vdash \Delta, A]$.

But still this is just an extensional match!

Steinberger's point stands:
what we want from entailment is some permissive, positive status.

Implicit assertion provides this,
using only the raw materials of positions and disagreement.

How this is connected to consequence
is a question about how disagreement works.