# Position-theoretic semantics and entailment

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# Position-theoretic semantics

#### The first central notion of this talk is the position.

# A position $[\Gamma \succ \Delta]$ is any pair of sets of sentences, with $\Gamma$ the sentences asserted and $\Delta$ the sentences denied.

(Sentences, assertion, denial, all taken for granted here.)

We track each other's evolving positions in conversation, and our conversational moves depend on this.

('No, I didn't eat it' is only an appropriate response to someone who asserted that you ate it.)

#### The second central notion is disagreement between positions.

Positions *P* and *Q* disagree when one person's adopting *P* and another's adopting *Q* would constitute a disagreement between those people.

> I'll write  $P \frown Q$ to indicate P disagrees with Q.

### Some boring notation:

# Subposition: $[\Gamma \succ \Delta] \sqsubseteq [\Sigma \succ \Theta] \quad \text{iff}_{df} \quad \Gamma \subseteq \Sigma \text{ and } \Delta \subseteq \Theta$

Position union:  $[\Gamma \succ \Delta] \sqcup [\Sigma \succ \Theta] =_{df} [\Gamma \cup \Sigma \succ \Delta \cup \Theta]$ 

### Some more interesting notation:

#### Self-disagreement:

$$[\Gamma \vdash \Delta] \quad \text{iff}_{df} \quad [\Gamma \succ \Delta] \frown [\Gamma \succ \Delta] \qquad \qquad P^{\vdash} \quad \text{iff}_{df} \quad P \frown P$$

# Little positions: + $\phi =_{df} [\phi \succ] - \phi =_{df} [\succ \phi]$

# Disagreement range / equivalence: $\hat{P} =_{df} \{R|P \frown R\}$ $P \simeq Q \quad \text{iff}_{df} \quad \hat{P} = \hat{Q}$

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Assumption 1: monotonicity
If P \sqsubseteq P' and Q \sqsubseteq Q' and P \frown Q,
then P' \frown Q'.
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It follows that if  $P \frown Q$ , then  $(P \sqcup Q)^{\vdash}$ .

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Assumption 2: disaggregation
If (P \sqcup Q)^{\vdash}, then P \frown Q.
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# With both assumptions in place, we can reduce disagreement to self-disagreement:

# $P \frown Q$ iff $(P \sqcup Q)^{\vdash}$

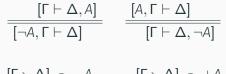
This is of technical convenience, but shouldn't be overstated.

It is disagreement is directly tied to conversational actions, and we might want to question these assumptions.

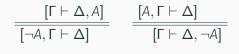
# We can build a compositional semantics in these terms, rather than, eg, truth, falsity, warrant, inference, etc.

Work by way of assertion and denial conditions, understood as conditions under which assertions and denials disagree.

$$\frac{[\Gamma \vdash \Delta, A]}{[\neg A, \Gamma \vdash \Delta]} - \frac{[A, \Gamma \vdash \Delta]}{[\Gamma \vdash \Delta, \neg A]}$$

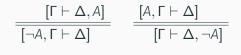


$$\frac{[\Gamma \succ \Delta] \frown \neg \neg A}{[\Gamma \succ \Delta] \frown \neg \neg A} = \frac{[\Gamma \succ \Delta] \frown \neg \neg A}{[\Gamma \succ \Delta] \frown \neg \neg A}$$



$[\Gamma \succ \Delta] \frown -A$	$[\Gamma \succ \Delta] \frown + A$
$[\Gamma \succ \Delta] \frown + \neg A$	$[\Gamma\succ\Delta]\frown\neg\neg A$

$$\widehat{+\neg A} = \widehat{-A} \qquad \widehat{-\neg A} = \widehat{+A}$$



$[\Gamma \succ \Delta] \frown -A$	$[\Gamma \succ \Delta] \frown + A$
$[\Gamma \succ \Delta] \frown + \neg A$	$[\Gamma \succ \Delta] \frown \neg\neg A$

$$\widehat{+\neg A} = \widehat{-A} \qquad \widehat{-\neg A} = \widehat{+A}$$

$$-A \simeq + \neg A + A \simeq - \neg A$$

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \land B, \Gamma \vdash \Delta]} \quad \frac{[\Gamma \vdash \Delta, A] \quad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \land B]}$$

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \land B, \Gamma \vdash \Delta]} \quad \frac{[\Gamma \vdash \Delta, A] \quad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \land B]}$$
$$\frac{[\Gamma \succ \Delta] \frown [A, B \succ]}{[\Gamma \succ \Delta] \frown -A} \quad \frac{[\Gamma \succ \Delta] \frown -B}{[\Gamma \succ \Delta] \frown -A \land B}$$

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \land B, \Gamma \vdash \Delta]} = \frac{[\Gamma \vdash \Delta, A] \quad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \land B]}$$

$$\frac{[\Gamma \succ \Delta] \frown [A, B \succ]}{[\Gamma \succ \Delta] \frown +A \land B} = \frac{[\Gamma \succ \Delta] \frown -A}{[\Gamma \succ \Delta] \frown -A \land B}$$

$$\widehat{+A \land B} = \widehat{[A, B \succ]} = \widehat{-A \land B} = \widehat{-A \cap -B}$$

$$\frac{[A, B, \Gamma \vdash \Delta]}{[A \land B, \Gamma \vdash \Delta]} = \frac{[\Gamma \vdash \Delta, A] \quad [\Gamma \vdash \Delta, B]}{[\Gamma \vdash \Delta, A \land B]}$$
$$\frac{[\Gamma \succ \Delta] \frown [A, B \succ]}{[\Gamma \succ \Delta] \frown (-A \land B)} = \frac{[\Gamma \succ \Delta] \frown (-A \land B)}{[\Gamma \succ \Delta] \frown (-A \land B)}$$
$$\widehat{+A \land B} = \widehat{[A, B \succ]} = \widehat{(-A \land B)} = \widehat{(-A \land B)}$$

 $+A \land B \simeq [A, B \succ]$ 

# What's key in this approach is just which positions disagree.

Models can show non-disagreement, but the models aren't the point.

Proofs can show disagreement, but the proofs aren't the point.

If disagreement reduces to self-disagreement, then everything is settled by ⊢.

The formal tools of most direct use are thus consequence-theoretic.

Consequence relations themselves matter, not any particular way of determining them.

Can this do what we want a semantics to do?

#### Dowty, Wall, Peters 1981:

"In constructing the semantic component of a grammar, we are attempting to account...[for speakers'] judgements of synonymy, entailment, contradiction, and so on" (2, emphasis added).

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#### Dowty, Wall, Peters 1981:

"In constructing the semantic component of a grammar, we are attempting to account...[for speakers'] judgements of synonymy, entailment, contradiction, and so on" (2, emphasis added).

# A problem: entailment

# There might seem to be an easy path to entailment: say that $\Gamma$ entails A iff $[\Gamma \vdash A]$

If some controversial assumptions hold, I'll argue, this view is extensionally right.

But even if this is so, it's more or less a coincidence.

### Steinberger (2011):

"Take the example of the classical theoremhood of the law of the excluded middle. [This] would have to be rendered as 'It is incoherent to deny  $A \lor \neg A$ '. But surely this is not what is intended; even the intuitionist can happily agree that it is incoherent to deny (every instance of)  $A \lor \neg A$ . [We need] a way of expressing that  $A \lor \neg A$  can always be correctly asserted" (353).

 $[\Gamma \vdash A] \text{ just says:} \\ [\Gamma \succ A] \text{ disagrees with itself.}$ 

If anything, it's a prohibition, a ruling out. But entailment should enable us to go on.

In the limiting case of empty  $\Gamma$ , it should ensure that *A* is assertible.

 $[\vdash A]$  doesn't do that.

#### Or: if truth and falsity are projections from assertion and denial,

then  $[\Gamma \vdash A]$  just says:  $\Gamma$  can't be true while A is false.

#### But entailment should connect truth to truth.

In the limiting case of empty  $\Gamma$ , it should ensure that A is true.

 $[\vdash A]$  doesn't do that.

# A solution: implicit assertion

# Recall that $P \simeq Q$ iff $\hat{P} = \hat{Q}$

As far as disagreements go, equivalent positions are just the same.

This means an adopter of *P* has the same options open for going on as an adopter of *Q* does.

# Say that a position *P* implicitly asserts A iff: $P \simeq P \sqcup +A$

# A is implicitly asserted when adding a genuine assertion of A wouldn't add any new disagreements.

(Mutatis for implicit denial, but that won't play a role here.)

### Implicit assertion is a broad notion, subsuming assertion proper.

# $([A, \Gamma \succ \Delta] \sqcup + A \text{ is just } [A, \Gamma \succ \Delta] \text{ again.})$

### Implicit assertion can help us fill the role of entailment.

### Write $P \Vdash^+ A$ to indicate that P implicitly asserts A.

If  $P \Vdash^+ A$ , when someone has already at least adopted P, they change nothing by going on to assert A. Implicit assertion is monotonic:

If  $P \sqsubseteq Q$  and  $P \Vdash^+ A$ , then  $Q \Vdash^+ A$  too.

Proof sketch:

Always  $\widehat{Q} \subseteq \widehat{Q \sqcup + A}$ , so we only need to show the converse.

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Suppose, then, that Q \sqcup +A \frown R.
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Since P \sqsubseteq Q, Q is P \sqcup S for some S, so P \sqcup S \sqcup + A \frown R.
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This means P \sqcup + A \frown R \sqcup S.
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But *P* implicitly asserts *A*, so  $P \frown R \sqcup S$ .

Finally, this gives  $P \sqcup S \frown R$ , which is to say  $Q \frown R$ .

# Implicit assertion has other structural properties we might expect of entailment.

For any A, we have  $+A \Vdash^+ A$ 

If  $P \Vdash^+ A$  and  $+A \sqcup P \Vdash^+ B$ , then  $P \Vdash^+ B$  Suppose our earlier theory of conjunction's assertion conditions:  $\widehat{+A \land B} = \widehat{[A, B \succ]}.$ 

Then  $[A, B \succ] \Vdash^+ A \land B$ .

Here's why:

Suppose  $[A, B, A \land B \succ] \frown R$ ; that is, that  $+A \land B \sqcup [A, B \succ] \frown R$ .

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Then +A \wedge B \frown R \sqcup [A, B \succ].
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By the assertion conditions,  $[A, B \succ] \frown R \sqcup [A, B \succ]$ .

And so  $[A, B \succ] \frown R$ .

Any position asserting A and B implicitly asserts  $A \wedge B$ .

#### Or suppose that $[\succ] \Vdash^+ A$ .

If  $[\succ]$  implicitly asserts A, then every position does.

If this is so, assertions of A are free; they close off no options at all, for anyone.

> This is just what we were after: a permissive notion.

# Entailment and consequence

Hmmm

Getting to  $[A, B_{\succ}] \Vdash^+ A \land B$  assumed only  $+A \land B = [A, B_{\succ}]$ . plus monotonicity and disaggregation.

> This is not enough to show  $[A, B \vdash A \land B]$ ; we'd want in addition that  $[A \land B \vdash A \land B]$ , which does not follow from anything so far.

(Entailment is reflexive, but consequence may not be!)

So we don't have an implication from  $[\Gamma \succ \Delta] \Vdash^+ A$  to  $[\Gamma \vdash \Delta, A]$ 

#### Hmmm

There is also no implication in general from  $[\Gamma \vdash \Delta, A]$  to  $[\Gamma \succ \Delta] \Vdash^+ A$ .

Suppose A is such that  $+A \frown [\Gamma \succ \Delta]$  and  $-A \frown [\Gamma \succ \Delta]$ . (Maybe  $[\Gamma \succ \Delta]$  has it that A attributes a vague predicate to one of its borderline cases.)

And suppose there is some Q with  $[\Gamma \succ \Delta] \not \sim Q$ .

Then  $[\Gamma \vdash \Delta, A]$  but  $[\Gamma \succ \Delta] \not\Vdash^+ A$ .

#### So in general $[\Gamma \vdash \Delta, A]$ and $[\Gamma \succ \Delta] \Vdash^+ A$ are independent claims.

However, if disagreement obeys certain properties, then these collapse.

# If $[A \vdash A]$ and $[\Gamma \succ \Delta] \Vdash^+ A$ , then $[\Gamma \vdash \Delta, A]$ .

If you'd self-disagree by denying and asserting A, then you'd self-disagree by denying A if you've implicitly asserted it.

# If $[\Gamma \vdash \Delta, A]$ and $[A, \Gamma \vdash \Delta]$ implies $[\Gamma \vdash \Delta]$ ,

and  $[\Gamma \vdash \Delta, A]$ , then  $[\Gamma \succ \Delta] \Vdash^+ A$ .

If disagreeing with both +A and -A means self-disagreeing, then if you disagree with -A you've implicitly asserted it.

# So if we assume $\vdash$ obeys identity and cut, then we get $[\Gamma \succ \Delta] \Vdash^+ A$ iff $[\Gamma \vdash \Delta, A]$ .

#### But still this is just an extensional match!

#### Steinberger's point stands: what we want from entailment is some permissive, positive status.

# Implicit assertion provides this, using only the raw materials of positions and disagreement.

How this is connected to consequence is a question about how disagreement works.