## Plumwood on classical and relevant logics

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## Val Plumwood

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### Val Plumwood (1939–2008) was an Australian philosopher best known for her work on ecofeminism.



She made significant contributions to logic, and for this received the title of Lady Plumwood in the Logicians' Liberation League.



# Her best-known contribution to logic is probably the "Routley star".

This provides a modal treatment of negation in relevant logic.

It was published in 1972 by Plumwood and Richard Sylvan (both then with the surname Routley)

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### A star frame contains a set of worlds and a unary operation $\star$ on worlds such that for each world *w*, we have $w^{\star\star} = w$ .

Conjunction and disjunction at each world is usual, but negation has a twist:  $w \Vdash \neg \phi \text{ iff } w^* \not\vDash \phi$ 

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Other modal vocabulary (boxes, diamonds, arrows, whatevs) can also be handled on star frames, by adding appropriate relations on worlds.

Logical interactions between negation and this other vocabulary are then handled by frame conditions tying **\*** to those relations.

This became known as the 'Australian plan'; it is the standard way to give world-based models for relevant logics.

# If we stick to just $\land, \lor, \neg$ , star frames give the logic FDE.

Example FDE validities:  $\neg (p \land q) \vdash \neg p \lor \neg q$  $\neg \neg p \vdash p$ 

Example FDE invalidities:  $p \land \neg p \nvdash q$  $\nvdash p \lor \neg p$ 

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## Dichotomy and dualism

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There are at least three strains of feminist theory that attack the practice of formal logic as problematically patriarchal.

### Feminist critics of logic have targeted at least: logic's authority, logic's abstraction, and logic's treatment of negation.

For example, Nye (1990) takes aim at authority:

"The relations between speakers that logic structures are alien to feminist aims...[T]he point of logic is to frame a way of speaking in which what another says does not have to be heard or understood, in which only the voice of a unitary authority is meaningful...[N]o application of logic can be feminist."

#### Hart (1993) worries about abstraction:

"[L]ogic is a good servant but a bad master. Its pulling back from primary experience is extremely valuable when relativized as a tool but becomes damaging and oppressive when pulled out of its proper context, hypostatized, and absolutized."

#### And Jay (1981) focuses on the role of negation:

"[M]en and women are conceived of in ways that cannot be a consequence only of conceptualization and reinforcement of empirical distinctions between them. Concepts of femaleness and maleness come into being that have nothing whatever to do with human sexual differences, but follow from the nature of contradictory dichotomy itself." Plumwood engages this tradition in her 1993 "The politics of reason: Towards a feminist logic".

This piece has two targets:

feminist philosophers who reject formal logic altogether, and philosophers of logic who see the choice between logical systems as apolitical.

Plumwood is concered to rebut a picture of logic that she sees these targets as sharing.

Plumwood:

"The construction of logic as a monolith, in which undiscriminating types of feminist critique collude [with 'establishment histories'], is precisely what has permitted formal logical systems and principles to be considered value free and to escape serious social criticism or examination." If there's only one way to do logic, then doing logic in that way can't be a political choice. The only choice is whether or not to do logic at all.

Plumwood, by contrast, wants to see particular logical systems as bearing signs of social power and perhaps of resistance to that power.

#### The picture:

Dominant social structures select and promote certain 'theories and technologies', which in turn promote and reinforce these structures (and that's in part why they were selected)

Plumwood locates classical logic among such 'theories and technologies'

#### Plumwood focuses on classical negation in particular.

As she sees it, Jay-style concerns are correct, but only about classical logic.

Plumwood's favoured logics—relevant logics are importantly different.

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"[P]erspectives naturalising an account of the other in terms of dualism and domination have had a great deal to do with which principles and accounts of negation have been viewed as 'normal', 'intuitive', and worthy of investigation and teaching, and which have been viewed as 'deviant' and of formal or specialist interpretation only."

#### Plumwood distinguishes *dichotomy* from *dualism*.

Dichotomy is simple non-identity or difference, and held innocent.

Dualism, by contrast, is much more involved, and is where Plumwood focuses her arguments.

When power is exercised systematically, it comes with dualistic ideology: dependence of the dominant on the subordinate is denied, and the subordinate are understood as inferior.

Examples: male/female, mind/body, civilised/primitive, human/natural.

Plumwood uses the terms 'master'/'other' to speak about such dualisms generally.

# Plumwood outlines four features common to dualisms, and argues that each is mirrored in classical negation.

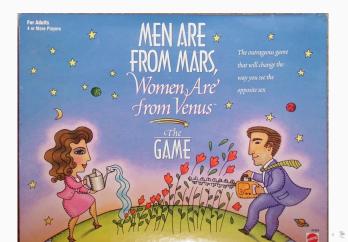
#### Two are captured neatly by Frye:

"To make [domination] seem natural, it will help if it seems to all concerned that members of the two groups are very different from each other, and this appearance is enhanced if it can be made to appear that within each group, the members are very like one another. In other words, the appearance of the naturalness of the dominance of men and the subordination of women is supported by anything which supports the appearance that men are very like other men and very unlike women, and that women are very like other women and very unlike men."

Features of dualism

Plumwood calls these 'hyperseparation' and 'homogenization':

Cross-group commonalities and intra-group differences are removed, reduced, denied, and ignored.



### Hyperseparation and homogenization create the appearance of two separate orders of being, two separate natures.

#### The remaining features of dualism break the symmetry.

### Relational definition: the other is understood by their difference from the master, not vice versa.

de Beauvoir "[M]an defines woman not in herself but as relative to him...She is defined and differentiated with reference to man and not he with reference to her"

Henrietta vs Susanissimo

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Backgrounding:

all the real actions, goals, desires, intentions are the master's; the other is just stage-setting, unacknowledged.

If the other has ends at all, these are understood in relation to the master's.

the Bechdel test

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## Classical vs relevant negation

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Classical negation, Plumwood argues, does not merely register dichotomy, but also exhibits these features of dualism.

In the classical relation between p and  $\neg p$ , Plumwood sees echoes of relational definition, hyperseparation, and homogenization.

(Plumwood connects backgrounding to implication rather than negation; I leave it aside)

# Futher, Plumwood claims that relevant negation does *not* exhibit these features.

In what follows, I'll explore what Plumwood might mean by holding that classical negation exhibits these features, and explore whether FDE negation exhibits them as well. Relational definition: "In classical logic [¬p] is interpreted as the universe without p, everything in the universe other than what p covers...¬p can then not be independently or positively identified, but is entirely dependent on p for its specification."

The negation of FDE is offered as a contrast, as allowing for  $\neg p$  to be 'independently characterized and with an independent role on its own behalf'. In classical worlds models, which worlds  $\neg p$  is true at is fully determined by which worlds p is true at.

But the reverse is also true; which worlds *p* is true at is fully determined by which worlds ¬*p* is true at.

So this is unlikely to be what Plumwood means.

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Plus, in a star frame, the same is true: which worlds either of *p* or ¬*p* holds at is fully determined by which worlds the other holds at.

# Perhaps the key is in the idea that $\neg p$ can't be 'positively identified'.

For example, there is no sentence (in usual vocabularies) classically equivalent to  $\neg p$  that does not use negation or falsum.

But the same is true of FDE. (Strictly *fewer* equivalences hold in FDE.)

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#### In classical logic, negation is *congruential*: $A \dashv \vdash B$ implies $\neg A \vdash \neg B$

## This is true of FDE as well, but not of many richer relevant logics.

 $p \rightarrow p \text{ and } q \rightarrow q$ 

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## But this is not a good understanding either, since it again does not distinguish between positive and negative.

#### Classically, $\neg A \dashv \vdash \neg B$ also implies $A \vdash B$

# The trouble with connecting classical logic to relational definition seems to be in double negation principles.

Whatever holds in general between A and ¬A will also hold in general between ¬A and ¬¬A, and so to the extent that A and ¬¬A can be exchanged for each other it will hold between ¬A and A.

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Perhaps the point is about uniqueness?

In a language with two negations, if both exhibit all the features of classical negation, the two negations are equivalent to each other.

This is not true of FDE negation; many nonequivalent FDE negations can live in the same language.

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### On this reading, once we have a certain relevant negation in view, the key difference disappears.

# The point is just that there were multiple relevant negations to choose from.

This isn't really about any particular negations, but about the category 'relevant negations'

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#### Classically, $\neg p$ is 'indistinguishable from the rest of the universe'.

Once we have the boundary drawn by p, the boundary drawn by  $\neg p$  is no further boundary.

This is not true of FDE negation; things can differ in whether they are  $\neg p$ without also differing in whether they are p

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But this is hardly an improvement.

Moving from two homogenized categories to four homogenized categories is not liberation from homogenization.

### Note that the Routley star as standardly presented comes with a restriction: that $w^{**} = w$ .

This ensures double negation properties, and together with usual theories of ∧, ∨ gives us De Morgan negation.

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Think, though, about modal logic K or relevant logic B.

Once frame-based tools are developed, it's often useful to consider them *unrestricted*.

### If we use the Routley star unrestricted, we arrive at *Ockham negation*.

Keep De Morgan principles, drop double negation principles.

Now none of  $p, \neg p, \neg \neg p, \neg \neg p, \ldots$  are equivalent

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### This would seem to address Plumwood's worry about homogenization, and in a Plumwoodian way

This is a lovely negation.

It is not the negation of any usual relevant logic.

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#### Plumwood sees explosion $p \land \neg p \vdash q$ as an echo of hyperseparation.

FDE indeed does not validate explosion.

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#### Plumwood

'Semantically, p and  $\neg p$  are treated classically as maximally distant in situational space. The extreme penalty classical logic provides for conjoining p and 'its' other not-p establishes a maximally strong relation of exclusion between p and  $\neg p$ , in comparison to other systems of propositional logic which define much weaker exclusion relationships'

#### What is 'maximally distant in situational space'? Plumwood doesn't say.

My best guess: the distance between two propositions is a measure of how many circumstances they disagree at.

With classical worlds,

this makes the distance between p and  $\neg p$  always 1. Not so in star frames, where this distance can vary freely.

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#### To connect this to hyperseparation, I think it's best to interpret points in these frames as properties and p and $\neg p$ as second-order properties.

For example, they might outline which properties are manly vs womanly, mental vs physical, colonial vs native, etc

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#### Then the distance between p and $\neg p$ is a measure of how many properties are pish without being $\neg p$ ish, and vv

## Classical negation then holds that any pish property must not be $\neg p$ ish, and vv

FDE allows for more flexibility.

This does seem to connect to hyperseparation, but by introducing an unusual interpretation of frames.

Is classical logic really selected on the basis of this interpretation?

### Conclusion

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- Plumwood holds that classical logic's position as 'normal' is in part because of its alignment with dualistic ideology.
  - This includes relational definition, homogenization, hyperseparation, and backgrounding.
  - She also holds that relevant logic does not so align.

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- But classical negation does not seem to align with relational definition; it's too symmetric.
- Concerns about homogenization seem to suggest Ockham negation, a different negation altogether.

• There may be a connection to hyperseparation, but seemingly only through a nonstandard interpretation.

#### Plumwood's attempt to defend formal logic by focusing criticism on classical logic only doesn't work as it stands.

The differences between classical and relevant logics are not as important as she claims.