Liar, curry, sorites: how many paradoxes?

David Ripley

Monash University
http://davewripley.rocks

The question

Eubulides listed 7 paradoxes:

the liar, the hooded man, the Electra, the overlooked man,
the heap, the bald man, and the horns

These seem ‘really’ to be four:
the hooded man, the Electra, and the overlooked man are the same,
as are the heap and the bald man

Because the heap and the bald man are the same paradox,
it's no good to solve them two different ways.

Colyvan:

"[I]t would be unacceptable to deal with the paradox of the heap by invoking a multivalued logic, $Ł_\infty$, say, and yet, when faced with the paradox of the bald man, invoke a supervaluational logic."

This also underpins revenge reasoning:

a solution that says one thing to the liar,
but another to a trivial variant on the liar,
is no good.

One attempt to ceremonialize this kind of thing:

Priest (BTL, p. 183)

"[I]t is natural to expect all the paradoxes of a certain family to have a single kind of solution. Any solution that can handle only some members of the family is bound to appear not to have got to grips with the fundamental issue. Let us call this the Principle of Uniform Solution (PUS): same kind of paradox, same kind of solution."

So far so good. But which paradoxes are of the same kind?

Priest's answer (and mine):
paradoxes are of the same kind
when they have the same explanation.

Liar, curry, sorites:
one, two, or three?

(And if two, which two?)

Liar

This sentence is not true.

Curry

If this sentence is true, then Sweden deserved to win Eurovision.

Sorites

H_0 is not a heap.

If something is not a heap, adding one grain will not make a heap.

So $H_{10^{10}}$ is not a heap.

Inclosure

Some have tried to unify sorites and the liar,
via the idea of an 'inclosure paradox':

Priest:

"[A]t a fundamental level, [the sorites and the liar] are the same. Both are inclosure paradoxes...The two kinds of paradox must therefore have the same kind of solution."

Weber et al:

"[T]he sorites paradox is an inclosure paradox, of a kind with [the liar.]"

An inclosure paradox is one whose explanation
is that it appears to instantiate the inclosure schema.

Skipping the schema for time reasons;
but I grant that the liar and sorites do appear to instantiate it.

Curry is not an inclosure paradox;
it doesn’t at all appear to instantiate the schema.

(The schema is tied firmly to negation, which curry doesn’t use.)

This then gives us two kinds among our three paradoxes:
liar and sorites on the one hand,
and curry on the other.

This division fits well with the formal approaches
of Priest, Weber, *Spandrels*-era Beall.

Wait a minute...

The liar and the curry have some obvious similarities:
truth, non-wf-reference, downward-entailing contexts.

Soriteses have none of these.

Plus, we can use these similarities to generate more paradoxes:

- At most two of these sentences are true.
- At most two of these sentences are true.
- At most two of these sentences are true.

This suggests a different division:
one where the liar and curry go together,
and the sorites is something else.

What does this have to do with explanation, though?

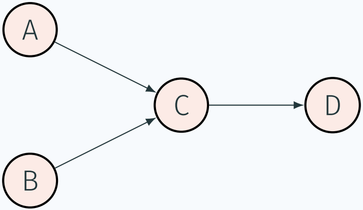
Explanation

I'll assume a crude counterfactual picture of explanation.

A first pass:

F explains *G* iff: if *F* hadn't obtained, *G* wouldn't have obtained

Problem: backtracking counterfactuals.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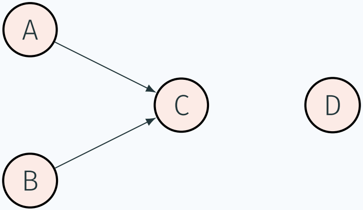
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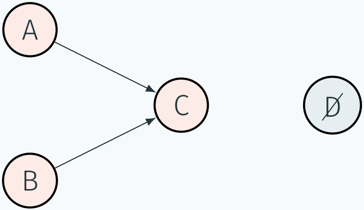
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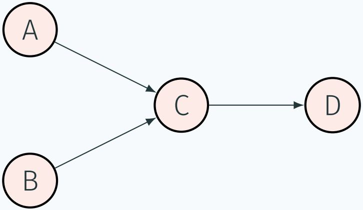
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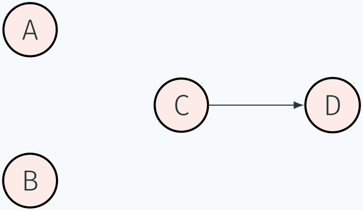
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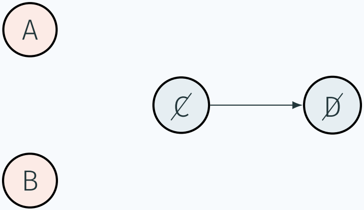
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If such-and-such wasn't the case, the liar wouldn't be paradoxical.

If such-and-such wasn't the case, the curry wouldn't be paradoxical.

If such-and-such wasn't the case, the sorites wouldn't be paradoxical.

Intervening on paradoxes

Priest wants to use PUS to argue for dialetheism,
so he needs to classify on his way to a solution.

I'll do the reverse:
assume a certain solution, and use that to classify.

All of these paradoxes involve failures of cut.

If they instead obeyed cut,
there would be no paradox.

But there's more to say.

Let's start by stepping into the mirror:

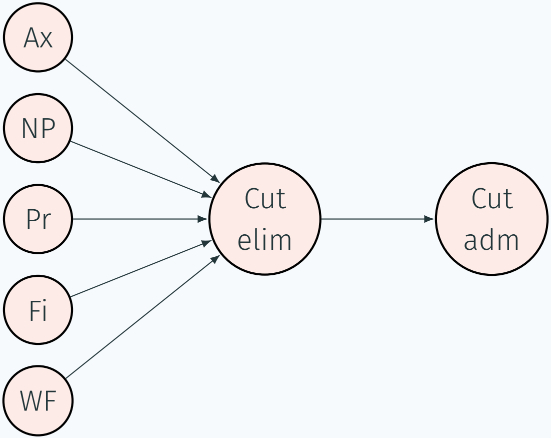
What explains cut admissibility when it is admissible?

Claim: cut elimination in an appropriate proof system.

Okay, but then what explains cut elimination?

Three key moves in the Gentzen-Bimbó proof,
supported by two key facts about the proof system in play.

Ax	Cut on axioms is idle.
NP	Cut on nonprincipals could have been done earlier.
Pr	Cut on principals can be replaced by cuts on their components.
Fi	Proof branches are finite.
WF	The component relation is wf.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Cut elim
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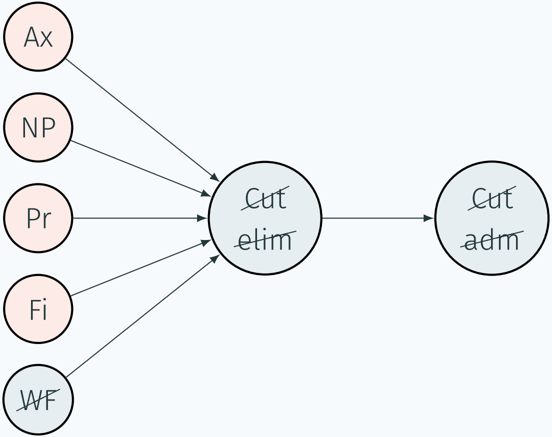
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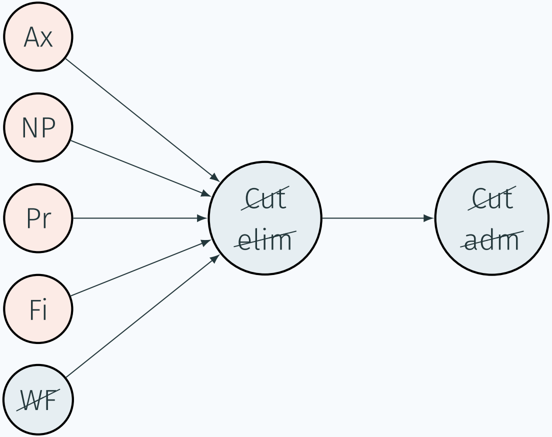
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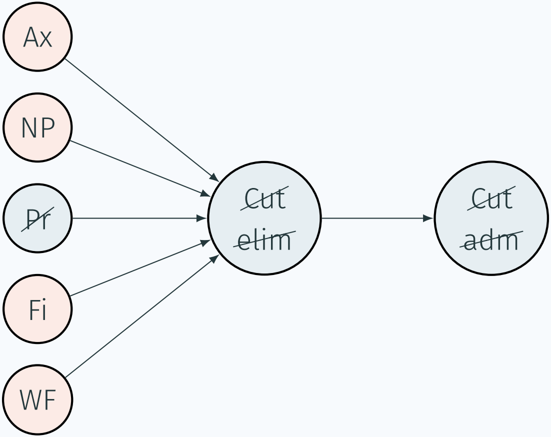
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If the component relation were well-founded,
the liar and curry would not be paradoxical.

If principal cuts could be pushed up,
the sorites would not be paradoxical.

- Counting paradoxes is a matter of seeking explanations.
- ‘Inclosure paradoxes’ gives one attempt at this, linking liar & sorites, and separating curry.
- Explanatory cut-elimination proofs give another, linking liar & curry, and separating sorites.
- Confirmation of what we all knew, really.

Inclosures

The inclosure schema has two ingredients subject to two conditions.

The ingredients:

- Ω A set
- δ A partial function $\wp(\Omega) \nrightarrow \Omega$

The conditions:

- $\delta(\Omega)$ is defined
- $\delta(X) \notin X$

These conditions are contradictory.

$\delta(\Omega) \in \Omega$, since $\delta(\Omega)$ is defined and δ's codomain is Ω.

But $\delta(\Omega) \notin \Omega$, since $\delta(X) \notin X$.

To cast the liar into the inclosure schema:

The ingredients:

• Ω	A set	The set of true sentences in $\mathcal{L}$
• δ	A p.f. $\wp(\Omega) \nrightarrow \Omega$	$X \mapsto$ 'This sentence is not in X.'

($\delta(X)$ is defined when X is definable in $\mathcal{L}$.)

The conditions:

•	$\delta(\Omega)$ is defined	'The true sentences' defines Ω.
•	$\delta(X) \notin X$	If it was, it wouldn't be true.

$\delta(\Omega)$ is the liar sentence.

To cast the sorites into the inclosure schema,
fix a given sorites series for the predicate P.

The ingredients:

- Ω A set — The set of P things in the series
- δ A p.f. $\wp(\Omega) \nrightarrow \Omega$ — $X \mapsto$ the next thing in the series beyond X

($\delta(X)$ is total, by tolerance.)

The conditions:

- $\delta(\Omega)$ is defined — δ is total.
- $\delta(X) \notin X$ — By defn.

$\delta(\Omega)$ is the first non-P thing in the series.