Dialogue and structural rules

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Overview

Catarina Dutilh Novaes has long argued for the centrality of dialogue in thinking about deductive reasoning.

Her recent book *The Dialogical Roots of Deduction* is a wide-ranging treatment of this topic.

Part of this book, drawing on earlier joint work with Rohan French, addresses substructural logics, while centering the role of dialogue.

I find this work fascinating, and have learned a fair bit from it.

Nonetheless, this talk is basically me having a whinge about their methodology.

Dialogue and deduction

By 'deduction',
Dutilh Novaes means an unusual activity
with a specific cultural history,
pursued by trained specialists.

It's what mathematicians do when proving something,
not just ordinary careful reasoning.

On the historical side,
Dutilh Novaes traces the history of deduction
to Aristotle and Euclid.

She explicitly sets aside early Indian
and Chinese logical traditions as not deductive.

Drawing on cognitive science,
Dutilh Novaes paints deduction as an activity
that most people simply don't do,
and one engaged in only rarely even by people who do do it.

Well-known difficulties in training advanced mathematics students
to produce mathematical proofs are very much to the fore.

Dutilh Novaes argues that this specific practice
is and has been crucially tied to dialogue.

She develops a notion of a Prover-Skeptic game,
a kind of quasi-adversarial language game,
to limn the relevant notion of dialogue.

"Prover wants her proof to go through no matter what..., regardless of whether it is a valid proof or not. Skeptic, by contrast, wants valid proofs to go through and invalid ones to be refuted, and is neutral with respect to 'pay-offs' of the game... Prover can win or lose the game, and Skeptic can neither win nor lose"

Sometimes this game, or something like it, is actually played:
for example between author and referee in mathematics journals.

Mostly, though, deductions are constructed
by someone playing the game internally,
taking the roles of both Prover and Skeptic.

I, at least, have found it helpful and interesting
to look at deductive reasoning through this lens.

I'm suspicious of the explicitly Eurocentric history,
but I think this is relatively separable from the rest.

Dutilh Novaes's approach strikes me as at least somewhat fair
to my own experiences of constructing proofs.

I find it especially useful for thinking about ways of teaching logic.

Substructural logics

Let's think now about formal logics.

I'll suppose we have some formal language,
and that arguments have the form Γ/A,
where Γ is a multiset of premises from the language
and A is a single conclusion from the language.

A logic, as I'll use the term, is then any set of arguments.

This is a very thin conception of logics:

logics in my sense are humble mathematical objects,
the same kind of thing that numbers are (whatever that is).

They carry no commitments at all
about truth, necessity, form, good reasoning,
or anything else people might associate with the word 'logic'.

Sometimes, people use logics in this thin sense
to make claims about truth, necessity, form, good reasoning, etc.

For example, someone might hold up a logic and say something like,
"All the arguments in this logic are necessarily truth-preserving!"

or

"To reason correctly, every inferential transition you make
must be in this logic!"

But each logic itself
is separate from any such claims.

Someone might also hold up a logic and say something like,
"This can be mapped on to certain electrical circuits!"

or

"Every theorem of this logic corresponds to a computable function!"

And of course, logics can also be studied on their own,
without making any claims like these examples.

Different logics differ from each other
in terms of the arguments they contain.

There are three ways to categorize logics
that are relevant here.

Some logics contain every argument with the form A/A;
these are called reflexive.

Others are missing some of these arguments,
and so are nonreflexive.

Some logics are such that,
whenever an argument $\Gamma, A, A/B$ is in them,
then $\Gamma, A/B$ is also in them.

These logics are called contractive,
and the others noncontractive.

Finally, some logics are such that,
whenever arguments Γ/A and A/B are both in them,
then Γ/B is also in them.

These logics are called transitive,
and the others nontransitive.

Very many logics that people study and apply
are all of: reflexive, contractive, and transitive.

Logics missing some of these features are among the
substructural logics.

Substructural logics, like any logics,
can be studied for pure curiosity's sake,
even though modern universities might want you to forget that.

But they've also been proposed as useful
for many different applications.

Here are some examples from the logical literature on paradoxes like the liar paradox 'This sentence is not true'.

Some people have proposed irreflexive logics
for thinking about reasoning under supposition.

Think about Γ/A as saying, 'if someone correctly supposes everything in Γ,
they can correctly conclude A under this supposition'.

Then if some C cannot be correctly supposed at all,
we should be interested in logics that exclude C/C.

But why think that anything cannot be correctly supposed?

Perhaps paradox: maybe the liar paradox is too ill-behaved
even to be supposable.

Others have proposed noncontractive logics
for thinking about states of affairs.

Think about Γ/B as saying,
'if the combination of the states of affairs in Γ obtains,
then necessarily the state of affairs B obtains'

If there is an A that yields something different when combined with itself,
then we should be interested in logics
that can contain $A, A/B$ without also containing A/B.

But why think any state of affairs is like this?

Perhaps paradox: on some views, the liar undergoes
constant atemporal self-annihilation.

'the liar is not true' gives rise to 'the liar is true', and vice versa, endlessly.

This is only contradictory if we can somehow get two at once!
Otherwise, the paradoxical state of affairs gets out of its own way.

Still others have proposed nontransitive logics
for thinking about constraints on the speech acts of assertion and denial.

Think about Γ/A as saying,
'it's forbidden to assert everything in Γ while denying A'.

If there is some B that it's forbidden both to assert and to deny,
then we should be interested in nontransitive logics for this interpretation,
since both Γ/B and B/A will always hold for such a B,
no matter what Γ and A are; but still Γ/A might not hold.

Why think that anything is forbidden both to assert and to deny?

Perhaps paradox: the very reasoning that shows the liar to be a problem
rules out asserting it and rules out denying it.

This is just a sampling.

Substructural logics are like other logics in this regard:
there's a wide range of applications people have proposed for them, and
only some have anything to do with paradox.

Because logics themselves are so thin and abstract,
they are open to multiple divergent interpretations.

Even when we focus on paradox, though,
the above interpretations of arguments diverge.

On the example nonreflexive approach,
arguments tell us about correct supposition and reasoning.

On the example noncontractive approach,
arguments tell us about states of affairs and their combinations.

And on the example nontransitive approach,
arguments tell us about constraints on speech acts.

There is no phenomenon X, then, where
the nonreflexive partisan takes X to be nonreflexive,
the noncontractive partisan takes X to be noncontractive,
and the nontransitive partisan takes X to be nontransitive.

Advocates of these different theories
are talking about different phenomena.

Two consequences:

Advocates of different substructural approaches
don't necessarily disagree with each other!

And where there is disagreement,
it's often about which phenomena to focus on.

The Dutilh Novaes / French argument

Dutilh Novaes and French bring a dialogical approach
to this debate about paradoxes.

But they do not enter into the debate as participants;
they instead aim to enter as referees.

"[D]ialogical interpretations of structural rules... provide a conveniently neutral framework to adjudicate between the different substructural proposals that have been made in the literature on paradoxes."

Here is their method:
they propose an application of arguments to Prover-Skeptic games.

They interpret an argument Γ/A as saying,
"If a participant in a Prover-Skeptic game grants everything in Γ,
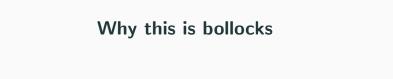then they must, if challenged, grant A as well"

Here, 'granting' and 'challenging' are moves in the game,
and 'must' has the force of a game rule.

Taking up this interpretation, they then argue
that transitivity is a nonnegotiable requirement,
that contraction is at least to be expected,
and that reflexivity is more or less useless.

They thus conclude that nonreflexive approaches to paradox
are better positioned than noncontractive approaches,
which in turn outperform nontransitive approaches.

As they put it,

"Insofar as one is sympathetic to the dialogical perspective, such a discussion may provide reasons to prefer one substructural solution over others... [T]his ranking [that is, nonreflexive > noncontractive > nontransitive] defines an order in which substructural solutions to paradoxes are to be preferred over others, from a dialogical perspective."

Why this is bollocks

Dutilh Novaes & French claim to take a 'dialogical perspective'
on existing substructural approaches to paradox.

This is not really what's happening, though.

Instead, they've created Frankenstein-monster-style theories,
taking the logics people have put forward with various interpretations,
and substituting their own dialogical interpretation instead.

That's not a perspective on the original theory;
it's just a different theory!

For example, take a nontransitive theory.

Some have claimed that the logic of assertion and denial is nontransitive,
because some things are forbidden both to assert and to deny.

The notion of 'forbidden' in question
comes from the linguistic norms that give our words their meanings.

Nobody has ever claimed, to my knowledge,
that this has anything to do with Prover-Skeptic games,
or even with deductive reasoning in Dutilh Novaes's narrow sense.

So even if we grant Dutilh Novaes & French
that the logic of Prover-Skeptic games must be transitive,
this tells us nothing about existing nontransitive theories of paradox.

The same goes for noncontractive theories.

Nobody has ever claimed, to my knowledge,
that the logic of Prover-Skeptic games is noncontractive.

Arguing that it is contractive, then, does not help us
evaluate existing noncontractive theories.

The same even goes for nonreflexive theories,
which Dutilh Novaes & French are claiming to defend.

Their defense is not of the existing nonreflexive theories,
but of a new theory based on Prover-Skeptic games instead.

What, then, do the arguments of Dutilh Novaes & French accomplish?

As far as I can see, they put three new theories of paradox on the table.

All are based on Prover-Skeptic games,
and they differ in terms of the logical structure
they ascribe to such games.

That seems like a great thing to do!

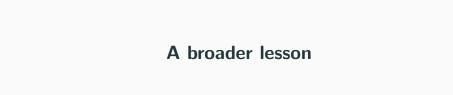What it is not, though, is a way of evaluating existing theories.

Instead of adjudicating the debate,
they have instead taken up three new positions in it.

A broader lesson

Here are two questions philosophers sometimes ask:

Which logic should we use to understand paradoxes?

How should we interpret the logic we use to understand paradoxes?

Different theories of paradoxes answer these questions in different ways.

Our example nontransitive theory says:
"a nontransitive logic, interpreted as about speech acts"

Our example noncontractive theory says:
"a noncontractive logic, interpreted as about states of affairs"

This might look like there are two points of disagreement:
one about which logic to use, and another about how to understand it.

However, there is no reason to think
that we should expect only one logic to be useful
in understanding paradoxes (or anything else).

(There are so-called 'logical monists',
but they don't think anything this silly!)

So there need be no disagreement
between advocates of these theories.

Maybe we should use a nontransitive logic for speech acts
and a noncontractive one for states of affairs.

Why not?

The kinds of things that are reflexive or not,
contractive or not, transitive or not, are logics.

A logic on its own is just a mathematical object;
it makes no claims at all.

The claims logicians make about paradoxes, then
always, necessarily, go beyond just giving a logic.

Typically, they involve giving a logic
together with an interpretation of that logic.

It is not sensible to evaluate such a claim
by substituting a different interpretation.

The takehome:

When people make arguments for or against some particular logic,
it is always with some interpretation at least implicitly in mind.

These arguments can't be evaluated
by changing the subject to some other interpretation.

Appendix: The argument for transitivity

Here, in full, is Dutilh Novaes & French's argument
that their dialogical notion of consequence must be transitive:

"[F]rom a dialogical perspective it seems very difficult to formulate
acceptable reasons to restrict Transitivity. Indeed, it is hard to see how a
proponent of a dialogical conception of logic, in terms of the
Prover-Skeptic dialogues we have been developing, could possibly justify
parting with it. None of the reasons adduced by proponents of
nontransitive solutions seem to apply to these specific dialogues."

Of bloody course 'None of the reasons adduced by proponents
of nontransitive solutions seem to apply to these specific dialogues'!

This is what I've been at pains to emphasize:
moving the conversation to Prover-Skeptic dialogues is
changing the subject.

But:

Recall that Dutilh Novaes & French read an argument
Γ/A as saying that a participant
who has granted everything in Γ
must also grant A, if challenged to do so.

These Prover-Skeptic games help us understand the process
of Prover leading Skeptic through a proof.

Here's a familiar phenomenon:
after Skeptic grants certain premises Γ,
Prover asks Skeptic to grant B as well.

Skeptic refuses at first.
But Prover then points out that Skeptic must grant A,
and, having granted A, must grant B.

This can all be fully in order!

So the logic of Prover-Skeptic games is nontransitive.