"Classical logic is not so much wrong—if by this word we mean that it ascribes disputable properties to the logical constants it deals with—as ambiguous: its connectives are ill-defined inasmuch as they have multiple meanings" (Paoli 2007)

"There is no need to give up any compelling inferential principle of classical logic—only to recognize that bad things can happen when principles holding of different connectives are used, in the course of a derivation, as holding of the same ambiguous connective…" (Mares & Paoli 2014)

"[W]e do not maintain that there are *invalid* inferential principles in classical propositional logic; if properly disambiguated, …all laws of classical logic can be salvaged in LL" (Mares & Paoli 2014)

Recapture via ambiguity

Atoms At	$:= p, q, r$, etc.	
Classical ($\mathcal{L}_C$) C	$:= At \mid \neg C \mid \mathbb{1}$	$\mid C \wedge C$
Linear ($\mathcal{L}_L$) $L :=$	$:= At \mid \neg L \mid \top \mid \mathsf{t}$	$\mid L \sqcap L \mid L \otimes L$

Other connectives can be defined in usual classical ways with negation: in $\mathcal{L}_C : \vee, \rightarrow, \mathbb{0}$; and in $\mathcal{L}_L : \sqcup, \parr, \sqsupset, \multimap, \bot, \mathsf{f}$.
All sequents are multiset-multiset.

Classical and linear languages

Id: $\dfrac{}{[p \succ p]}$ $\boxed{\boxed{\text{D: } \dfrac{[\Gamma \succ \Delta]}{[\Gamma', \Gamma \succ \Delta, \Delta']}}}$ $\neg$L: $\dfrac{[\Gamma \succ \Delta, A]}{[\neg A, \Gamma \succ \Delta]}$ $\neg$R: $\dfrac{[A, \Gamma \succ \Delta]}{[\Gamma \succ \Delta, \neg A]}$

$\sqcap$L: $\dfrac{[A/B, \Gamma \succ \Delta]}{[A \sqcap B, \Gamma \succ \Delta]}$ $\sqcap$R: $\dfrac{[\Gamma \succ \Delta, A] \quad [\Gamma \succ \Delta, B]}{[\Gamma \succ \Delta, A \sqcap B]}$ $\otimes$L: $\dfrac{[A, B, \Gamma \succ \Delta]}{[A \otimes B, \Gamma \succ \Delta]}$ $\otimes$R: $\dfrac{[\Gamma \succ \Delta, A] \quad [\Gamma' \succ \Delta', B]}{[\Gamma, \Gamma' \succ \Delta, \Delta', A \otimes B]}$

$\top$R: $\dfrac{}{[\Gamma \succ \Delta, \top]}$ tL: $\dfrac{[\Gamma \succ \Delta]}{[\mathsf{t}, \Gamma \succ \Delta]}$ tR: $\dfrac{}{[\succ \mathsf{t}]}$

Two systems: AL is the full system; LL is AL minus D. Cut's admissible in both.

Affine logic AL and linear logic LL

Id: $\dfrac{}{[p \succ p]}$ $\boxed{\boxed{\text{D: } \dfrac{[\Gamma \succ \Delta]}{[\Gamma', \Gamma \succ \Delta, \Delta']}}}$ $\neg$L: $\dfrac{[\Gamma \succ \Delta, A]}{[\neg A, \Gamma \succ \Delta]}$ $\neg$R: $\dfrac{[A, \Gamma \succ \Delta]}{[\Gamma \succ \Delta, \neg A]}$

$\boxed{\wedge\text{L}^{\sqcap}\text{: } \dfrac{[A/B, \Gamma \succ \Delta]}{[A \wedge B, \Gamma \succ \Delta]}}$ $\wedge$R$^{\sqcap}$: $\dfrac{[\Gamma \succ \Delta, A] \quad [\Gamma \succ \Delta, B]}{[\Gamma \succ \Delta, A \wedge B]}$ $\wedge$L$^{\otimes}$: $\dfrac{[A, B, \Gamma \succ \Delta]}{[A \wedge B, \Gamma \succ \Delta]}$ $\boxed{\wedge\text{R}^{\otimes}\text{: } \dfrac{[\Gamma \succ \Delta, A] \quad [\Gamma' \succ \Delta', B]}{[\Gamma, \Gamma' \succ \Delta, \Delta', A \wedge B]}}$

$\mathbb{1}$R$^{\top}$: $\dfrac{}{[\Gamma \succ \Delta, \mathbb{1}]}$ $\mathbb{1}$L$^{\mathsf{t}}$: $\dfrac{[\Gamma \succ \Delta]}{[\mathbb{1}, \Gamma \succ \Delta]}$ $\boxed{\mathbb{1}\text{R}^{\mathsf{t}}\text{: } \dfrac{}{[\succ \mathbb{1}]}}$

Four systems: G3C$^+$ is the full system; G3C* is G3C$^+$ minus the single-box rules;
NOD is G3C$^+$ minus D; NOD$^-$ is just the totally unboxed rules.

G3C$^+$ and G3C* have the same derivable rules;
both determine classical logic.

Systems for the classical language

Grishin (γ) and Ono (o) translations ($\mathcal{L}_{\mathrm{C}}$ to $\mathcal{L}_{\mathrm{L}}$)

C	$\gamma^-(C)$	$\gamma^+(C)$	$o^-(C)$	$o^+(C)$
p	p	p	$\mathsf{t} \sqcap p$	$\mathsf{f} \sqcup p$
$\mathbb{1}$	t	$\top$	t	$\top$
$\neg A$	$\neg\gamma^+(A)$	$\neg\gamma^-(A)$	$\neg o^+(A)$	$\neg o^-(A)$
$A \wedge B$	$\gamma^-(A) \otimes \gamma^-(B)$	$\gamma^+(A) \sqcap \gamma^+(B)$	$o^-(A) \otimes o^-(B)$	$o^+(A) \sqcap o^+(B)$

Old fact (Grishin): $\Gamma \vdash_{\mathrm{CL}} \Delta$ iff $\gamma^-[\Gamma] \vdash_{\mathrm{AL}} \gamma^+[\Delta]$.
Old fact (Ono): $\Gamma \vdash_{\mathrm{CL}} \Delta$ iff $o^-[\Gamma] \vdash_{\mathrm{LL}} o^+[\Delta]$.

L	p	$\top$	t	$\neg B$	$B \sqcap C$	$B \otimes C$
$\beta(L)$	p	$\mathbb{1}$	$\mathbb{1}$	$\neg\beta(B)$	$\beta(B) \wedge \beta(C)$	$\beta(B) \wedge \beta(C)$

Proof of Grishin fact: apply $\gamma^\pm$ to a G3C* derivation of $[\Gamma \succ \Delta]$, get an AL derivation of $[\gamma^-[\Gamma] \succ \gamma^+[\Delta]]$;
apply β to an AL derivation of $[\gamma^-[\Gamma] \succ \gamma^+[\Delta]]$, get a G3C* derivation of $[\Gamma \succ \Delta]$.

Translations from $\mathcal{L}_{\mathrm{C}}$ to $\mathcal{L}_{\mathrm{L}}$

Problem 1: Why mess with the atoms?
Problem 2: What does ambiguity have to do with $+/-$ occurrences?

The Ono translations don't fit the ambiguity story

In response to the first worry, (Mares & Paoli 2014) suggests using $\gamma^\pm$ with LL:

> "[I]f we confine ourselves to the classical tautologies that play a role in the known versions of the paradoxes, you do not need to replace propositional variables in order to get to a theorem of LL"

They 'recapture' the logic $\Vdash$ such that $\Gamma \Vdash \Delta$ iff $\gamma^-[\Gamma] \vdash_{\mathrm{LL}} \gamma^+[\Delta]$.

New fact: This is the system determined by NOD$^-$. It's not classical. For example, $p \wedge q \nVdash p$. (*Proof* just as Grishin fact.)

The Grishin translations and linear logic

On a better version of the story, 'speakers' of $\mathcal{L}_{\mathrm{C}}$ do not really speak ambiguously: *both* meanings are always present.
Classical connectives *run together* distinct meanings. This is *conflation*.

(Ripley 2017) offers a formal treatment of conflation, there called *blurring*:
an argument is valid in a conflated language iff it has some valid *unconflated counterpart*.
Two key desiderata support this: *inferential charity* and *minimality*.

In this case, β gives exactly what is needed: a map from the unconflated language to the conflated language.
A $\mathcal{L}_{\mathrm{C}}$ argument A should be valid iff there is some valid underlying $\mathcal{L}_{\mathrm{L}}$ argument mapped to A by β.

Ambiguity and conflation

New fact: $\Gamma \vdash_{\mathrm{CL}} \Delta$ iff there are Γ', Δ' such that: $\Gamma' \vdash_{\mathrm{AL}} \Delta'$ and $\beta[\Gamma'] = \Gamma$ and $\beta[\Delta'] = \Delta$.

New fact: $\Gamma \vdash_{\mathrm{NOD}} \Delta$ iff there are Γ', Δ' such that: $\Gamma' \vdash_{\mathrm{LL}} \Delta'$ and $\beta[\Gamma'] = \Gamma$ and $\beta[\Delta'] = \Delta$.

Proof of these facts: apply β to an AL/LL derivation of $[\Gamma' \succ \Delta']$, get a G3C$^+$/NOD derivation of $[\beta[\Gamma'] \succ \beta[\Delta']]$;
and for any G3C$^+$/NOD derivation of $[\Gamma \succ \Delta]$, *there is some way* to pull it apart into an appropriate AL/LL derivation.
(Each connective occurrence in $[\Gamma \succ \Delta]$ is introduced once in the derivation; choose its sharpening depending on how.)

Recapture via blurring

	LL	AL
$\gamma^\pm$	NOD$^-$	G3C*
β	NOD	G3C$^+$

The difference between LL and AL is D.
The difference between $\gamma^\pm$ and β is the boxed rules.

The Mares & Paoli story as given does not fit the surrounding sales pitch: it recaptures NOD$^-$, not classical logic.
Even the improved version I've offered here only gets to NOD.
Classical recapture via these routes looks to require D.

Summary