Axioms $A \rhd A$, where A is atomic

KL: $\dfrac{\Gamma \rhd \Delta}{\Gamma, A \rhd \Delta}$ KR: $\dfrac{\Gamma \rhd \Delta}{\Gamma \rhd A, \Delta}$ WL: $\dfrac{\Gamma, A, A \rhd \Delta}{\Gamma, A \rhd \Delta}$ WR: $\dfrac{\Gamma \rhd A, A, \Delta}{\Gamma \rhd A, \Delta}$

Cut: $\dfrac{\Gamma \rhd A, \Delta \qquad \Gamma', A \rhd \Delta'}{\Gamma, \Gamma' \rhd \Delta, \Delta'}$

Structural rules

$\neg$L: $\dfrac{\Gamma \rhd A, \Delta}{\Gamma, \neg A \rhd \Delta}$ $\neg$R: $\dfrac{\Gamma, A \rhd \Delta}{\Gamma \rhd \neg A, \Delta}$ $\sqcap$L: $\dfrac{\Gamma, A_i \rhd \Delta}{\Gamma, A_0 \sqcap A_1 \rhd \Delta}$ $\sqcap$R: $\dfrac{\Gamma \rhd A, \Delta \qquad \Gamma \rhd B, \Delta}{\Gamma \rhd A \sqcap B, \Delta}$

$T\langle\,\rangle$L: $\dfrac{\Gamma, A \rhd \Delta}{\Gamma, T\langle A\rangle \rhd \Delta}$ $T\langle\,\rangle$R: $\dfrac{\Gamma \rhd A, \Delta}{\Gamma \rhd T\langle A\rangle, \Delta}$ $\otimes$L: $\dfrac{\Gamma, A, B \rhd \Delta}{\Gamma, A \otimes B \rhd \Delta}$ $\otimes$R: $\dfrac{\Gamma \rhd A, \Delta \qquad \Gamma' \rhd B, \Delta'}{\Gamma, \Gamma' \rhd A \otimes B, \Delta, \Delta'}$

Disjunctions $\sqcup$, $\oplus$ dual to $\sqcap$, $\otimes$, respectively.

Operational rules

MAAL:	Id + K + connective rules	MAALT:	MAAL + T rules
ST:	MAAL + W	STT:	ST + T rules = MAALT + W

Def: A sequent S is a *contraction* of a sequent S' iff: S can be derived from S' by applications of W. S' is an *expansion* of S; write $\widehat{S}$ for an expansion of S, and $\widehat{S}_1, \widehat{S}_2$, etc for more.

Def: For a set of sequents C, WC is the set of contractions of sequents in C, the closure of C under W.

Def: A derivation meets the W$\otimes$ (W$\oplus$) restriction iff: every application of $\otimes$R ($\oplus$L) has at least one premise-sequent derived without use of W.

Logics and definitions

MAALT $\subsetneq$ WMAALT $\subsetneq$ STT:
$\rhd p \sqcup \neg p$ is WMAALT-valid but not MAALT-valid.
$\rhd (p \sqcup \neg p) \otimes (p \sqcup \neg p)$ is STT-valid but not WMAALT-valid.
WMAALT is contractive, and has transparent truth:
it does not admit cut.

Theorem: WMAALT is STT with the W$\otimes$ and W$\oplus$ restrictions.
Corollary: WMAALT's additive fragment is STT.

Complete: A MAALT derivation followed by a bunch of Ws always meets the restrictions.

Sound: Transform restricted-STT derivation of a sequent S into MAALT derivation of some $\widehat{S}$.
Induction on size of restricted-STT derivation: base case obvious.

Contracting MAALT

$\sqcap$L: $\dfrac{\Gamma, A_i \rhd \Delta}{\Gamma, A_0 \sqcap A_1 \rhd \Delta}$ $\rightsquigarrow$ $\sqcap$L^n: $\dfrac{\widehat{\Gamma}, {A_i}^n \rhd \widehat{\Delta}}{\widehat{\Gamma}, A_0 \sqcap {A_1}^n \rhd \widehat{\Delta}}$

$\otimes$L: $\dfrac{\Gamma, A, B \rhd \Delta}{\Gamma, A \otimes B \rhd \Delta}$ $\rightsquigarrow$ $$\dfrac{\dfrac{\widehat{\Gamma}, A^n, B^m \rhd \widehat{\Delta}}{\widehat{\Gamma}, A^{\max(m,n)}, B^{\max(m,n)} \rhd \widehat{\Delta}}\ \text{KL}^{|n-m|}}{\widehat{\Gamma}, A \otimes B^{\max(m,n)} \rhd \widehat{\Delta}}\ \otimes\text{L}^{\max(m,n)}$$

Some one-premise rule cases

$$
{\otimes}\text{R: }\dfrac{\Gamma \triangleright A, \Delta \qquad \Gamma' \triangleright B, \Delta'}{\Gamma, \Gamma' \triangleright A \otimes B, \Delta, \Delta'}
\quad \rightsquigarrow \quad
\dfrac{\vdots}{\hat{\hat{\Gamma}}, \Gamma'^{n} \triangleright A \otimes B^{n}, \hat{\hat{\Delta}}, \Delta'^{n}}
$$

$$
{\otimes}\text{R: }\dfrac{{\otimes}\text{R: }\dfrac{{\otimes}\text{R: }\dfrac{\hat{\hat{\Gamma}} \triangleright A^{n}, \hat{\hat{\Delta}} \qquad \Gamma' \triangleright B, \Delta'}{\hat{\hat{\Gamma}}, \Gamma' \triangleright A^{n-1}, A \otimes B, \hat{\hat{\Delta}}, \Delta'} \qquad \Gamma' \triangleright B, \Delta'}{\hat{\hat{\Gamma}}, \Gamma'^{2} \triangleright A^{n-2}, A \otimes B^{2}, \hat{\hat{\Delta}}, \Delta'^{2}} \qquad \Gamma' \triangleright B, \Delta'}{\hat{\hat{\Gamma}}, \Gamma'^{3} \triangleright A^{n-3}, A \otimes B^{3}, \hat{\hat{\Delta}}, \Delta'^{3}}
$$

Restricted ⊗R case

$$
{\sqcap}\text{R: }\dfrac{{\sqcap}\text{R: }\dfrac{\text{KR: }\dfrac{\Gamma \triangleright A^{m}, \Delta}{\Gamma \triangleright A^{m}, B^{n-1}, \Delta} \qquad \text{KR: }\dfrac{\Gamma \triangleright B^{n}, \Delta}{\Gamma \triangleright A^{m-1}, B^{n}, \Delta}}{\Gamma \triangleright A^{m-1}, B^{n-1}, \Delta, A \sqcap B} \qquad \text{KR: }\dfrac{\Gamma \triangleright B^{n}, \Delta}{\Gamma \triangleright A^{m-2}, B^{n}, \Delta, A \sqcap B}}{\Gamma \triangleright A^{m-2}, B^{n-1}, \Delta, A \sqcap B^{2}}
$$

$$
\dfrac{\vdots}{\Gamma \triangleright B^{n-1}, \Delta, A \sqcap B^{m}}
$$

Similarly to reach $\Gamma \triangleright A^{m-1}, \Delta, A \sqcap B^{n}$.

⊓R case (subpart 1)

Part 1 starts from the pair of $\Gamma \triangleright A^{m}, \Delta$ and $\Gamma \triangleright B^{n}, \Delta$,
and gives both $\Gamma \triangleright A^{m-1}, \Delta, A \sqcap B^{n}$ and $\Gamma \triangleright B^{n-1}, \Delta, A \sqcap B^{m}$.
Use KR to align these to $\Gamma \triangleright A^{m-1}, \Delta, A \sqcap B^{\max(m,n)}$ and $\Gamma \triangleright B^{n-1}, \Delta, A \sqcap B^{\max(m,n)}$, and repeat.

Part 1 lets us remove one A from the 'A-sequent' and one B from the 'B-sequent',
at the price of adding lots of $A \sqcap B$s. KR lets us prepare the results
to be input into part 1 again. After $\min(m,n) - 1$ rounds of this,
we have either $\Gamma \triangleright A, \Delta, A \sqcap B^{x}$ or $\Gamma \triangleright B, \Delta, A \sqcap B^{x}$.

⊓R case (subpart 2)

$$
{\sqcap}\text{R: }\dfrac{\Gamma \triangleright A, \Delta \qquad \Gamma \triangleright B, \Delta}{\Gamma \triangleright A \sqcap B, \Delta}
$$

$$
\rightsquigarrow
$$

$$
\text{Part 2 }(\star)\text{: }\dfrac{\text{K: }\dfrac{\hat{\hat{\Gamma}}_1 \triangleright A^{m}, \hat{\hat{\Delta}}_1}{\hat{\hat{\Gamma}} \triangleright A^{m}, \hat{\hat{\Delta}}} \qquad \dfrac{\hat{\hat{\Gamma}}_2 \triangleright B^{n}, \hat{\hat{\Delta}}_2}{\hat{\hat{\Gamma}} \triangleright B^{n}, \hat{\hat{\Delta}}}}{\hat{\hat{\Gamma}} \triangleright A, \hat{\hat{\Delta}}, A \sqcap B^{x}}
$$

$$
{\sqcap}\text{R: }\dfrac{\text{KR: }\dfrac{\star\text{: }\dfrac{}{\hat{\hat{\Gamma}} \triangleright A, \hat{\hat{\Delta}}, A \sqcap B^{x}}}{\hat{\hat{\Gamma}} \triangleright A, B^{n-2}, \hat{\hat{\Delta}}, A \sqcap B^{x+1}} \qquad {\sqcap}\text{R: }\dfrac{\text{KR: }\dfrac{\hat{\hat{\Gamma}} \triangleright A, \hat{\hat{\Delta}}, A \sqcap B^{x}}{\hat{\hat{\Gamma}} \triangleright A, B^{n-1}, \hat{\hat{\Delta}}, A \sqcap B^{x}} \qquad \text{K: }\dfrac{\hat{\hat{\Gamma}}_2 \triangleright B^{n}, \hat{\hat{\Delta}}_2}{\hat{\hat{\Gamma}} \triangleright B^{n}, \hat{\hat{\Delta}}, A \sqcap B^{x}}}{\hat{\hat{\Gamma}} \triangleright B^{n-1}, \hat{\hat{\Delta}}, A \sqcap B^{x+1}}}{\hat{\hat{\Gamma}} \triangleright B^{n-2}, \hat{\hat{\Delta}}, A \sqcap B^{x+2}}
$$

$$
\dfrac{\vdots}{\hat{\hat{\Gamma}} \triangleright \hat{\hat{\Delta}}, A \sqcap B^{x+n}}
$$

⊓R case (main)