Containment and analyticity

David Ripley

Monash University
http://davewripley.rocks

Meaning containment (MC)

The idea behind the logic MC
is to use $\rightarrow$ to express meaning containment.

For example, $(A \wedge B) \to A$ is a theorem;
the meaning of $A \wedge B$ contains the meaning of A.

And $(A \to (A \to B)) \to (A \to B)$ is not;
that A contains the claim that A contains B
does not in general contain the claim that A contains B.

Indeed, even if it's just true that $A \to (A \to B)$,
it doesn't follow that $A \to B$;
A's containing the claim that A contains B
doesn't mean that A really does contain B.

If I hand you a card that says "Dave handed you a watermelon",
that doesn't mean I handed you a watermelon!

MC gives a framework for exploring this kind of claim.

Content models for MC work with sets of sentences in $\mathcal{L}$.
(This is a background language; it needn't be the propositional one!)

The content of a set of sentences is the set of sentences that can be analytically established from the initial set.

The 'content of' operation c is a closure on $\wp\mathcal{L}$.
We only look at closed sets, called contents.

Let $X \sqcup Y := c(X \cup Y)$.

Then we have a lattice of contents ordered by $\supseteq$, with $X \sqcup Y$ the glb/meet of X and Y and $X \cap Y$ the lub/join.

Interpret each propositional atom as a content,

let $[\![A \wedge B]\!]$ be $[\![A]\!] \sqcup [\![B]\!]$

and $[\![A \vee B]\!]$ be $[\![A]\!] \cap [\![B]\!]$.

On this picture, containment is literal set-theoretic containment.
$A \wedge B$ contains A because $[[A]] \sqcup [[B]] \supseteq [[A]]$.

To move to the higher-degree fragment, we need to ask after the contents of these set-theoretic containment claims themselves.

So we suppose that for contents C and D, the claim $C \supseteq D$ is itself a member of $\mathcal{L}$.

Moreover, suppose:

- $c(C \supseteq D) \sqcap c(D \supseteq E) \supseteq c(C \supseteq E)$
- $c(C \supseteq D) \sqcap c(C \supseteq E) \supseteq c(C \supseteq D \sqcap E)$
- $c(C \supseteq E) \sqcap c(D \supseteq E) \supseteq c(C \cap D \supseteq E)$

These build lattice properties into the content of these claims.

Finally, suppose that if $C \supseteq D$,
then $c(E \supseteq C) \supseteq c(E \supseteq D)$
and $c(D \supseteq E) \supseteq c(C \supseteq E)$.

This one in particular seems shady to me, but it's not my topic.

From this we get a nondistributive depth-relevant logic MC.

Features of the content models directly become theorems of MC.

eg:

$$((C \to D) \wedge (D \to E)) \to (C \to E)$$

$$c(C \supseteq D) \sqcap c(D \supseteq E) \supseteq c(C \supseteq E)$$

The focus here: $A \to (A \vee B)$.

$A \to (A \vee B)$ is a theorem of MC.

This is because $[[A \vee B]] = [[A]] \cap [[B]]$,
and $[[A]] \supseteq [[A]] \cap [[B]]$.

The set of sentences a.e. from a disjunction
is the intersection of the sets a.e. from the disjuncts.

So the content of a disjunct contains the content of the disjunction,
on this understanding of content.

Analytic containment

But there is another tradition with similar-sounding motivations.

This tradition rejects the containment of $A \vee B$ in A.

Say that C contains D iff C is synonymous with $C \wedge D$.

This is Angell's understanding, but in itself it's neutral.
Content models already give: $C \supseteq D$ iff $C = C \sqcup D$.

Angell (1989):

"S_1 entails S_2 only if the meaning of S_2 is contained in the meaning of S_1...S_1 is synonymous with S_2 iff S_1 entails S_2 and S_2 entails S_1. Taken together these dicta yield the familiar proposition that S_1 is synonymous with S_2 iff they contain all and only the same meanings" (119).

Again, this is all neutral so far.

Angell (1989):

"But it also seems obvious that in general S_1 will not contain all and only the same meanings as $S_1 \wedge (S_1 \vee S_2)$. To admit the principle of Absorption as a principle of entailment in our present sense, would be to say that two sentences could contain all and only the same meanings even though one referred to and talked about individuals the others [sic] did not, and/or used predicates the other did not" (121).

Angell's argument:

C is not in general synonymous with $C \wedge (C \vee D)$,
since D might talk about something C doesn't mention,
and so C does not contain $C \vee D$.

There are a bunch of so-called ‘containment logics’ stemming from work of Parry and Angell.

These mostly reject the claim that C contains $C \vee D$.

Take a domain D of objects and the set $\{\top, \bot\}$ of truth values.

Let each atom take a value $\langle C, V \rangle$
with C a subset of the domain and $V \in \{\top, \bot\}$.

- $\langle C, V \rangle \wedge \langle E, U \rangle = \langle C \cup D, V \wedge U \rangle$
- $\langle C, V \rangle \vee \langle E, U \rangle = \langle C \cup D, V \vee U \rangle$
- $\langle C, V \rangle \rightarrow \langle E, U \rangle = \langle C \cup D, X \rangle$

where $X = \top$ iff $E \subseteq C$ and $(V = \bot$ or $U = \top)$

Each sentence's value tracks which objects it's about.
$\rightarrow$s can only be true when their consequents
are about no more than their antecedents.

Comparing motivations

Brady (2006):

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Angell (2002):

"The conection of logical containment is…defined in terms of rigorous syntactic criteria for 'is logically synonymous with' " (xvii).

Angell (2002):

"For rigor, the rules for deriving theorems from axioms must be interpretable strictly as rules of syntactical transformations. But [all this] should be interpretable as conforming to the concept of referentially synonymous expressions ...

If two expressions are referentially synonymous, then 1) both expressions must refer to or talk about only the same entities, 2) both expressions must say (or predicate) the same things about each of those entities, and 3) all and only those things contained in (or entailed or implied) by one expression must be contained in (or entailed or implied) by the other" (36).

Whatever this is, it ain't syntactic!

Brady (2006) mischaracterises the difference.

It's not a question of syntactic vs semantic.

Everyone here is focused on meaning.

Entailment I:

"The system **PAI** is motivated by the idea that the consequent of an analytische Implikation should simply "unpack" the antecedent, and that in consequence such formulas as $A \to (A \lor B)$...should fail, since the consequents of these might refer to information not contained in the antecedents" (431).

Brady (2006):

"A sentence q is analytically established from a set X of sentences when q follows from the members of X by an analysis of the meanings of the various components of q and the members of x" (16).

Brady (2006):

"$p \vee q$ can be analytically established from p...This should be contrasted to Parry's analytic implication..., where the containment of conclusions in premises requires that no new variables are introduced at all. Our containment, on the other hand, relates the meanings of words in the conclusion with those from the premises" (19).

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The difference is about the kind of connection required for analytic validity:

must we unpack the premises alone to find the conclusion, or can we unpack premises and conclusion both to find the connection between them?

If the latter, we have $A \to (A \vee B)$; if the former, not.

Analytic validity

- The argument from p together with q to $p \wedge q$ is analytically valid.
- That argument turns on the meaning of $\wedge$, which appears just in the conclusion.
- So analytic validity can depend on the meanings of conclusion-only vocab.

So $p \vdash p \vee q$ gets to be analytically valid for the same reason as $p, q \vdash p \wedge q$.

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But seeing why it's too quick will not save the Parry/Angell approach.

In Parry/Angell-style sytems:
p together with q does indeed ‘contain’ $p \wedge q$.

But these logics can only see ‘p together with q’ as $p \wedge q$;
there is no notion of premise combination besides $\wedge$ itself.

The containment in question is $p \wedge q$’s containing itself.

This gives a principled way to resist the initial thought:

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But only by denying that $\wedge$I tells us anything about $\wedge$!

In Parry/Angell systems, p together with q contains $p \wedge q$, just because anything contains itself.

Brady's setting, by contrast, enables us to combine premises for analytic validity without $\wedge$.

This makes it possible to say that the content of p together with q is the content of $p \wedge q$, and mean this as a claim about $\wedge$.

Indeed, this is Brady's story of how $\wedge$ works in the content semantics.

Parry/Angell systems cannot even ask the question.

We could rig up some way for Parry/Angell systems
to combine antecedents without conjunction.

But this leaves a dilemma: does p with q contain $p \wedge q$ or not?

If so, then we're not just unpacking premise meanings.
Meanings in conclusions matter too.
We should allow that p contains $p \vee q$.

If not, then how is this meant to work?

- Both Brady's *MC* and Parry/Angell-style systems aim to capture the notion of content containment.
- A key difference is whether A contains $A \vee B$.
- We should understand analytic entailment as allowing appeal to meanings in the conclusion as well as the premise; otherwise we can't endorse $\wedge$I as a theory of $\wedge$.
- So on the best-motivated sense of containment, the meaning of A does contain the meaning of $A \vee B$.